Interaction of Josephson junction and distant vortex in narrow thin-film superconducting strips

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The phase difference between the banks of an edge-type planar Josephson junction crossing the narrow thin-film strip depends on wether or not vortices are present in the junction banks. For a vortex close to the junction this effect has been seen by Golod, Rydh, and Krasnov [Phys. Rev. Lett. **104**, 227003 (2010)], who showed that the vortex may turn the junction into π type. It is shown here that even if the vortex is far away from the junction, it still changes the 0 junction to a π junction when situated close to the strip edges. Within the approximation used, the effect is independent of the vortex-junction separation, a manifestation of the topology of the vortex phase which extends to macroscopic distances of superconducting coherence.

I. INTRODUCTION

It has long been known that Abrikosov vortices in the vicinity of Josephson junctions affect the junction properties [1]. Recent experiments with a vortex trapped in one of the banks of an edge-type planar junction in a thin-film superconducting strip showed that the vortex causes an extra phase difference on the junction that depends on the vortex position [2]. This effect is strong, in particular when the vortex is close to the junction; the situation when the junction character is changed from the conventional "zero"-type behavior to that of the π junction. In this communication we show that this can happen even if the vortex is far away from the junction but close to the strip edges. This is a manifestation of the phase coherence on macroscopic distances in superconductors and of the topology of the vortex phase. The effect can be utilized for manipulating Josephson currents by controlling the far-away vortex position.

II. PROBLEM

Consider a thin-film strip of a width W with an edge-type Josephson junction across the strip. The strip is narrow: $W \ll \Lambda = 2\lambda^2/d$, where λ is the London penetration depth of the film material and d is the film thickness. Choose x along the strip and y across it so that $0 < y < W$ and the junction is at $x = 0$. Let a vortex be pinned at $\boldsymbol{r}_0 = (x_0, y_0)$. The problem is to evaluate the phase of the vortex along the bank $x = +0$ of the junction, see Fig. 1.

The London equation integrated over the film thickness is

$$\frac{c}{2\pi\Lambda} h_z + \text{curl}_z\, \boldsymbol{g} = \frac{c\phi_0}{2\pi\Lambda}\,\delta(\boldsymbol{r} - \boldsymbol{r}_0), \qquad (1)$$

where h_z is the field component normal to the film and $\boldsymbol{g}(x,y)$ is the sheet current density. Since div $\boldsymbol{g} = 0$, one can look for $\boldsymbol{g} = \text{curl}\, S\hat{z}$, where $S(x,y)$ is the scalar stream function. For large Λ, the first term in Eq. (1) is small and we have

$$\nabla^2 S = -\frac{c\phi_0}{2\pi\Lambda}\,\delta(\boldsymbol{r} - \boldsymbol{r}_0). \qquad (2)$$

This is, in fact, a Poisson equation for a linear "charge" $c\phi_0/8\pi^2\Lambda$ at $\boldsymbol{r}_0$ so that the problem is equivalent to that in the two-dimensional (2D) electrostatics.

Moreover, the current is expressed either in terms of the gauge invariant phase φ or via the stream function S: $\boldsymbol{g} = -(c\Phi_0/4\pi^2\Lambda)\nabla\varphi = \text{curl}\, Sz$. This relation written in components shows that $(4\pi^2\Lambda/c\Phi_0)S(\boldsymbol{r})$ and $\varphi(\boldsymbol{r})$ are the real and imaginary parts of an analytic function.

The sheet current normal to the strip edges $y = 0, W$ is zero. In addition, one can disregard the Josephson tunneling currents relative to those of the vortex, i.e., setting $g_x(0,y) = 0$ as well. These boundary conditions imply that S is a constant along the edges of the half-strip; one can choose this constant as zero. Thus, the problem is formally equivalent to the 2D problem of the electrostatic potential S due to a linear charge at $\boldsymbol{r}_0$ on a half-strip with grounded edges, see the upper panel of Fig. 2.

III. CONFORMAL MAPPING

This problem can be solved by conformal mapping of the half-strip onto a half-plane for which the electrostatic field is easily found [3]. The relation

$$u + iv = -i\cosh\pi(x + iy) \qquad (3)$$

transforms the half-plane $u > 0$ to the half-strip of our interest as shown in Fig. 2. In real terms this transformation reads

$$u = \sinh\pi x\,\sin\pi y, \qquad v = -\cosh\pi x\,\cos\pi y. \qquad (4)$$

In particular, the vortex position (x_0, y_0) transforms to the point (u_0, v_0):

$$u_0 = \sinh\pi x_0\,\sin\pi y_0, \qquad v_0 = -\cosh\pi x_0\,\cos\pi y_0. \qquad (5)$$

The complex potential $F(u,v)$ for a linear charge (vortex) at $w_v = u_0 + iv_0$ at the half-plane $u > 0$ near the grounded plane at $u = 0$ is

$$F = 2q\ln\frac{w - w_v}{w - w_{av}} = 2q\left[\ln\frac{r_1}{r_2} + i(\theta_1 - \theta_2)\right], \qquad (6)$$

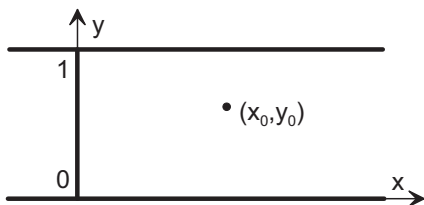where $w = u + iv$, and $w_{av} = -u_0 + iv_0$ is the position of the fictitious antivortex.

FIG. 1. The superconducting thin-film strip with a Josephson junction at $x = 0$ and a vortex at (x_0, y_0). The width W is used as a unit length.

$r_{1,2}$ and $\theta_{1,2}$ are the corresponding moduli and phases, see the lower panel of Fig. 2; q is the linear "charge" $c\phi_0/8\pi^2\Lambda$. Clearly,

$$
\begin{aligned}
r_1 &= \sqrt{(u - u_0)^2 + (v - v_0)^2}\,, \\
r_2 &= \sqrt{(u + u_0)^2 + (v - v_0)^2}\,,
\end{aligned}
\tag{7}
$$

and

$$
\theta_1 - \theta_2 = \tan^{-1}\frac{v - v_0}{u - u_0} - \tan^{-1}\frac{v - v_0}{u + u_0}\,.
\tag{8}
$$

The phase in the plane (x, y) is obtained by the substitution of Eqs. (4) and (5) in Eq. (8).

We are interested in the vortex phase at the junction bank $x = +0$, $0 < y < 1$, which corresponds to $u = +0$, $-1 < v < 1$ [4]:

$$
\begin{aligned}
\varphi(+0, y) &= -2\tan^{-1}\frac{v - v_0}{u_0} \\
&= 2\tan^{-1}\frac{\cos\pi y - \cosh\pi x_0\,\cos\pi y_0}{\sinh\pi x_0\,\sin\pi y_0}\,.
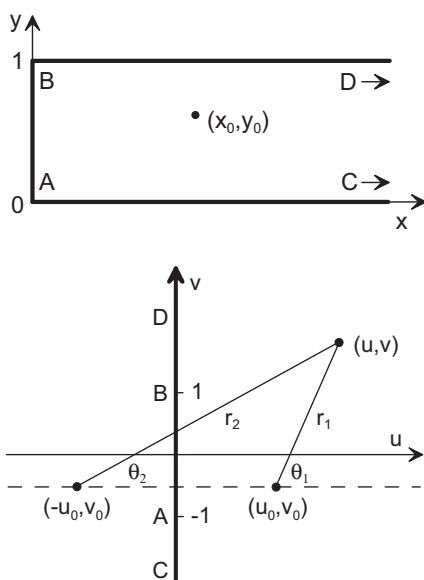\end{aligned}
\tag{9}
$$

FIG. 2. Upper panel: The half-strip of the width 1 with a vortex at (x_0, y_0). Equations (3) or (4) map the half-strip in the (x, y) plane onto the half-plane $u > 0$ (lower panel) with the vortex at (u_0, v_0); the points A, B, C, and D on both planes are shown.
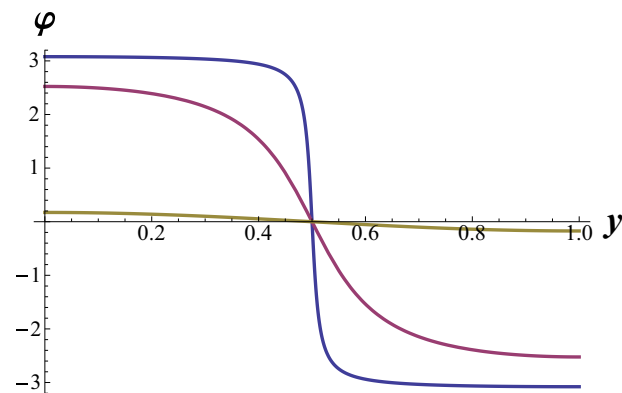

FIG. 3. (Color online) Contribution of a vortex at $(x_0, 1/2)$ to the phase at the right bank of the junction ($x = +0$) as a function of the coordinate y along the junction (in units of the junction length W). The curves are for $x_0 = 0.01,\ 0.1,\ 1$. For $x_0 = 1$, this contribution is close to zero and nearly constant.

For example, for a vortex in the strip middle $y_0 = 1/2$, we have

$$
\varphi(+0, y) = 2\tan^{-1}\frac{\cos\pi y}{\sinh\pi x_0}\,.
\tag{10}
$$

Hence, as is seen from Fig. 3, for the vortex close to the junction $x_0 \to 0$, the junction acquires the π character everywhere except in a narrow region of the fast changing vortex phase near y_0 [5]. It is seen from the figure that the vortex close to the junction induces the phase difference reminiscent of that of a Josephson vortex. In the later case, however, the vortex "core" or the region of the fast phase change is related to a fixed material parameter, the Josephson penetration depth, whereas in our case, as Eqs. (9) and (10) show, it is of the order x_0, the vortex-junction separation.

IV. FAR-AWAY VORTEX

It is of interest to examine what happens when the vortex is far from the junction. As follows from Eq. (9) for large x_0, the vortex contribution to the phase at the junction is a constant depending on the vortex position y_0:

$$
\varphi(+0, y) = \pi(2y_0 - 1) = C\,.
\tag{11}
$$

A remarkable new feature is worth noting: The vortex-junction separation x_0 drops from this result. A straightforward numerical check shows that the last equation approximates the full expression (9) for $x_0 > 2$ with an accuracy of less than 1%. Hence, the term "far-away" can be used reliably for vortices separated from the junction by more than $2W$.

When the vortex is in the strip middle $C = 0$, i.e., a far-away vortex does not affect the junction properties. When the vortex is close to the upper edge of the strip $C = \pi$, whereas for $y_0 \to 0$, the lower strip edge $C = -\pi$. Hence, when a far-away vortex approaches the strip edges, we in fact have a π junction. Moreover, any phase shift from 0 to π on the junction can be achieved by changing the vortex position y_0 from the middle to the strip edges. This remarkable topological property allows one to fine-tune the junction behavior by manipulating the positions of far-away vortices.

This, at first sight strange effect, can be understood by examining the current distribution for a vortex near the edge. When a vortex approaches the edge the current lines are squeezed so that the component g_x along the edge increases in a shrinking space between the vortex core and the edge. Formally, g_x diverges when $y_0 \to 0$, see the Appendix. Hence, the phase, the gradient of which is proportional to the current, varies very fast at the part of the edge adjacent to the vortex. On the other hand, the total phase change along any contour containing the vortex is 2π. Hence, when the vortex approaches the edge, nearly all available phase changes of 2π happen at the edge part adjacent to the vortex, being close to $-\pi$ on one side of this part and to $+\pi$ on the other. On the rest of the edge, the phase is nearly constant.

V. VORTEX IN A THIN FILM LOOP

Till now we considered an infinite strip with a junction and a vortex in one, e.g., the right half-strip. The vortex affects only the phase on the right junction bank. If, however, one has a closed superconducting thin-film loop, the vortex affects the phases on both junction banks. We do not have an exact solution for the vortex currents in this case. Still, one can use a qualitative argument to describe the situation as follows.

Consider a straight strip of a finite length $L \gg W$ with no junction and a vortex situated far from the ends $x = 0, L$. In principle, this problem can be solved by conformal mapping, but physically the picture should be similar to the half-infinite strip: According to Eq. (11) the phase at $x = 0$ will be $-\pi(1 - 2y_0)$ and at the end $x = L$ it is $\pi(1 - 2y_0)$ (as mentioned, the coordinate x_0 of the vortex does not enter these contributions).

We can now bend this long, but finite, strip to make a ring or loop so that the ends at $x = 0$ and $x = L$ come together to form a tunnel junction. The strip bending to a ring should have a small effect if $L \gg W$. Then the junction phase difference will be close to

$$\delta\varphi = -\pi(1 - 2y_0) - \pi(1 - 2y_0) = -2\pi + 4\pi y_0. \quad (12)$$

2π is irrelevant, hence $\delta\varphi = 4\pi y_0$. This gives $\delta\varphi = \pi$ for $y_0 = 1/4$ or $3/4$. Thus, when the vortex is near the edges $y_0 = 0, 1$ or in the strip middle with $y_0 = 1/2$, its contribution to the junction phase difference is zero, whereas for the vortex at $y_0 = 1/4, 3/4$ the junction acquires an extra phase difference of π. Hence, we expect that in a closed superconducting loop with a junction, a vortex at large distances from the junction still affects the junction properties, albeit differently from the case of infinite straight strips.

VI. DISCUSSION

The effect we describe is due to the topologic properties of the phase which extend to macroscopic distances of the superconducting coherence. As such it depends on the sample geometry. It should be stressed that in our derivation the connection between the phase difference on the Josephson junction and the vortex position is based on the assumption that the thin-film size is small relative to the Pearl length Λ. This allows us to disregard the magnetic field associated with vortex currents and to deal only with the kinetic part of the London energy. Formally, the problem becomes equivalent to those of 2D electrostatics and the conformal mapping technique can be employed. The three-dimensional effects in bulk materials are more complicated since magnetic fields have to be taken into account [1], a subject out of the scope of this paper.

Another property of the mesoscopic system considered here should be mentioned. Since the Josephson energy is $\propto (1 - \cos\delta\varphi)$ and the junction phase difference $\delta\varphi$ depends on the vortex position, the total energy of the system differs from the sum of the junction energy with no vortex and the vortex energy with no junction. In other words, there is the junction-vortex interaction which depends on the vortex position y_0 across the strip. Within the model considered, this interaction is of a purely topologic nature and does not depend on the junction-vortex separation.

APPENDIX: VORTEX NEAR STRIP EDGES

According to [6], the magnetic flux crossing the narrow strip due to a vortex at $(0, y_0)$,

$$\Phi_z = \frac{\phi_0}{\pi\Lambda}\left(y_0 \ln\frac{1 - y_0}{y_0} - \ln(1 - y_0)\right), \quad (A1)$$

vanishes as $-y_0 \ln y_0$ when the vortex approaches the edge $y = 0$. The question arises whether or not the sheet current $g_x(y_0 \to 0)$ at this edge vanishes as well.

The current distribution due to the vortex at $(0, y_0)$ is given by the stream function [6]

$$S = \frac{c\phi_0}{4\pi^2\Lambda}\tanh^{-1}\frac{\sin\pi y \ \sin\pi y_0}{\cosh\pi x - \cos\pi y \ \cos\pi y_0}. \quad (A2)$$

Consider $g_x = \partial S/\partial y$ at $x = 0$ and $0 < y < y_0$:

$$g_x = \frac{c\phi_0}{4\pi\Lambda}\frac{\sin\pi y_0}{\cos\pi y - \cos\pi y_0}. \quad (A3)$$

When $y_0 \to 0$ and so does $y < y_0$, we obtain

$$g_x \propto \frac{1}{y_0 - y} \to \infty. \quad (A4)$$

Hence, the phase varies very fast along the part of the edge adjacent to the vortex.

[1] M. V. Fistul and G. F. Giuliani, Phys. Rev. B **58**, 9343 (1998).

[2] T. Golod, A. Rydh, and V. M. Krasnov, Phys. Rev. Lett. **104**, 227003 (2010).

[3] According to the Schwarz-Christoffel theorem, this transformation exists since the half-strip is a triangle with one of the vertices at infinity; see, e.g., P. M. Morse and H. Feshbach, *Methods of Theoretical Physics* (McGraw-Hill, New York, 1953).

[4] J. R. Clem, Phys. Rev. B **84**, 134502 (2011).

[5] For a vortex close to the junction the phase change between $y = 0$ and $y = 1$ is 2π. According to the authors of Ref. [2] this change is π, the angle at which the junction is seen from the vortex position. This, however, would be the case only for the vortex in infinite film where the phase coincides with the azimuthal angle.

[6] V. G. Kogan, Phys. Rev. B **49**, 15874 (1994).