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Flux pinning mechanisms in type II superconductors

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Abstract

Expressions for flux-pinning in type II superconductors are derived from considerations of the nature of the interaction between individual flux-lines and pinning-centres, and of the geometry of the pinning-centres. It is demonstrated that the experimentally observed scaling laws are obtained without the necessity of introducing the concept of flux-lattice elasticity. Predicted pinning functions are found to provide adequate explanation of measured Lorentz force curves in a wide range of high-κ, strong-pinning materials.

§ 1. Introduction

Models which account for the critical current density and other reversible properties of type II superconductors are based on the concept of the critical state (Bean 1962, Kim, Hempstead and Strnad 1963). The critical current density J_c at an induction B is that which allows the Lorentz force to be just balanced by a pinning force F_p. The mixed state flux-line lattice is assumed to be pinned by interaction with pinning centres which are features of the microstructure of the superconductor. $F_p(B)$ is the strength of this interaction per unit volume actually being exerted at a given value of B, and is dependent upon B. The critical state may be represented by (Evetts, Campbell and Dew-Hughes 1968):

$$\bar{J}_c(B)\Lambda\bar{B} = -F_p(B). \tag{1}$$

The critical current density may be determined experimentally as a function of B (or more usually H; the difference between H and B in high-κ superconductors is negligible provided $H \gg H_{c1}$). The function $F_p(B)$ deduced from these measurements must be zero at $B=0$, pass through at least one maximum and return to zero at $B=\mu_0 H_{c2}$ where, by definition, $J_c=0$. Any realistic pinning mechanism must therefore predict a maximum in the Lorentz force.

If F_p is plotted as a function of the reduced field $h(=H/H_{c2})$ it is found that, for a given specimen, results at different temperature lie on one master curve (Fietz and Webb 1969). The pinning force can be represented by an expression of the form:

$$F_p(h) \propto H_{c2}{}^n(T) fn(h), \tag{2}$$

where the value of n, and the form of $fn(h)$ are characteristic of the pinning mechanism operating in the superconductor. Successful flux-pinning models must accurately predict the experimentally observed value of n and of $fn(h)$,

the most characteristic feature of the latter being the positions of any maxima in the curve. Several theories have tried to account for the observations in specific materials (see Dew-Hughes 1971 and Campbell and Evetts 1972, for a review of these). In this paper, an alternative approach to the problem of flux-pinning is presented, which is able to predict the various observed forms of $fn(h)$, depending upon the details of the pinning processes operative in a specific material.

§ 2. The pinning force

The pinning force per unit volume is given by

$$F_p = \eta L f_p = -\eta L \Delta W/x, \tag{3}$$

where f_p, the pinning force per unit length of pinned flux-line, is equal to ΔW, the work done in moving unit length of flux-line from a pinning centre to the nearest position where it is unpinned, divided by x, the effective range of the pinning interaction ; L is the total length of flux-line per unit volume that is directly pinned ; and η is an efficiency factor determined by the extent to which its neighbours in the flux-lattice allow a flux-line to relax toward a position of maximum pinning. These quantities are influenced by four factors :

(i) The superconducting nature of the pinning centres, as it is the difference between their superconducting properties and those of the matrix which determines the strength of the local interaction.

(ii) The size and spacing (or ' wavelength ') of the pinning microstructure compared to the superconducting penetration depth λ, as only if this is greater than λ can the local equilibrium value of B be established.

(iii) The size of the pinning centres compared with the flux-lattice spacing, as this determines the total length of interacting flux-line, and the geometrical nature of the interaction.

(iv) The rigidity of the flux lattice, as this determines whether displacements of flux-lines under local pinning forces are purely elastic, resisted by neighbouring flux lines, or the pinning forces are sufficiently strong to completely disrupt the lattice and allow each flux-line to act individually.

These are now considered in turn.

2.1. *Superconducting nature of the pinning centres*

Flux-lines interact with pinning centres because the superconducting properties of the latter are different from those of the bulk of the superconductor. The strength of the interaction is a function of the magnitude of this difference. The difference may be small, and manifest itself as a difference in critical temperature, critical field, or Ginsberg–Landau κ. The difference may be large, as is the case when the pinning centre is non-superconducting. The largest difference, and hence the strongest pinning, arises when the pinning centre is ferromagnetic (Alden and Livingston 1966, Campbell, Evetts and Dew-Hughes 1968). Of all the various possibilities,

only two are believed responsible for flux-pinning in the majority of commercial superconducting materials. These are (i) small differences in κ, arising from changes in the normal state resistivity, due to composition fluctuations (Witcomb and Dew-Hughes 1973), non-uniform distributions of dislocations (Dew-Hughes 1966), or martensite transformation (Dew-Hughes 1974), and (ii) non-superconducting particles, which may be normal metal, insulator or void. Bibby (1970), has shown that the pinning strength of non-superconducting particles is independent of the nature of the particles. If non-superconducting metallic particles have a diameter $\leqslant$ the superconducting coherence length ξ of the matrix, the proximity effect will induce them to become superconducting, and their presence can be regarded as producing a change in κ.

The two types of pinning centres give rise to what will subsequently be referred to as '$\Delta\kappa$ pinning' and 'normal pinning'.

2.2. *Wavelength of the microstructure*

Hampshire and Taylor (1972) pointed out the importance of wavelength of the microstructure relative to λ. The latter is the distance over which the induction, B, can undergo an appreciable change within the superconductor. If both the size, a, and spacing, l, of the pinning centres is greater than λ, B is able to adjust everywhere to its equilibrium value, different in the pins from that in the matrix. This difference gives rise to a Bean–Livingston barrier to flux motion at the interface between pin and matrix which is the cause of pinning (Campbell *et al.* 1968). This interaction will be referred to as the magnetic interaction.

When either a or l is less than λ, the induction cannot adjust to the local equilibrium value, and will assume some appropriate average value. The free energy of the flux lines now has a value in the pinning centres different from that in the matrix, due to the difference in superconducting properties. This will be referred to as the core interaction.

2.3. *Size of the pinning centres*

Pinning centres may be classified by the number of their dimensions which are large compared with the inter-flux-line spacing $d(=1{\cdot}07(\phi_0/B)^{1/2})$ (Campbell *et al.* 1968, Livingston 1968). Point pins are regions whose dimensions in all directions are less than d. A point pin can interact with only one flux-line at a time. Line pins, such as dislocations or needle-shaped precipitates, have one dimension long compared with d. When lying parallel to the local direction of B, they can interact with one flux line over their whole length. Lying at an angle to B they can interact with several flux lines. Grain-, twin- and martensite boundaries, stacking faults, dislocation arrays such as sub-grain and polygonized boundaries, plate-like precipitates, and the surface of the superconductor may have two dimensions greater than d and act as surface pins. They have the strongest influence when their plane normal is parallel to the direction of the Lorentz force. Volume pins, with all dimensions larger than d, are large precipitates and thick-walled dislocation cell-structures resulting from cold-deformation. Flux-lines interact with the surface of volume pins, and the important parameter is the projected area of surface in the direction of the Lorentz force.

2.4. *Flux-lattice rigidity*

Early models of flux-pinning ignored the inter-flux line forces, assuming by omission that the flux-line lattice was sufficiently relaxed to allow pinned flux lines to take up positions of minimum energy without restriction. Later theories (Labusch 1969, Fietz and Webb 1969) postulated that the lattice suffers elastic distortion under the action of pinning forces, but that no flux-line can move individually to a position of lowest energy. The distance moved by a flux-line will be the local pinning force divided by the appropriate elastic modulus, and the overall pinning strength will be reduced by the ratio of this distance to that through which the flux-line should have moved to become completely pinned.

The original lattice elasticity theory was derived for point pins ; it has since been extended to line pins (Campbell and Evetts 1972, Kramer 1973). It does not yet appear to have been applied to surface or volume pins, though modification to include such cases should be possible, requiring only a decision as to which are the appropriate elastic constants to be incorporated in Labusch's original equations.

§3. Pinning functions

With the foregoing, it is now possible to evaluate the quantities in eqn. (3) appropriate to various pinning situations. This is done in the following four sections, and the expressions as deduced are then incorporated into eqn. (3) to yield specific pinning functions.

3.1. *The work done per unit length of pinned line*, ΔW

ΔW depends upon the nature of the pinning interaction, that is whether it is magnetic or core pinning. The work done in moving a quantity of flux ϕ_0 through a boundary separating two regions of differing induction is $\phi_0 \Delta M(B)_{\mathrm{rev}}$, where $\Delta M(B)_{\mathrm{rev}}$ is the difference in reversible magnetization, in equilibrium with the local value of B, either side of the boundary (Campbell *et al.* 1968). $M(B)$ is assumed to be zero for non-superconducting regions ; any normal state paramagnetism is neglected. For non-superconducting pinning centres $\Delta M(B)_{\mathrm{rev}}$ is equal to the reversible, equilibrium, magnetization of the superconductor. Using Abrikosov's (1957) expression for this latter quantity :

$$\Delta W \text{ (mag, normal)} = \frac{-\phi_0(H_{c2}-H)}{1{\cdot}16(2\kappa^2-1)}. \tag{4}$$

$\Delta M(B)_{\mathrm{rev}}$ for $\Delta\kappa$ pinning centres is obtained by differentiating the above with respect to κ, remembering that H_{c2} is a function of κ, and ignoring the difference between κ_1 and κ_2 (Dew-Hughes and Witcomb 1972). For $\kappa \gg 1$,

$$\Delta W \text{ (mag, } \Delta\kappa) = \frac{-\phi_0(H_{c2}-2H)\Delta\kappa}{2{\cdot}32\kappa^3}. \tag{5}$$

When the pinning is due to a core interaction, ΔW is equal to Δg, the change in the Gibbs' function of unit length of flux-line as it moves from the pinning centre to the matrix. Δg is estimated following Hampshire and

Taylor (1972). The Gibbs' function per unit volume of a superconductor in the mixed state is (Goodman 1966) :

$$G_M(H, T) = G_N(H, T) - \frac{\mu_0(H_{c2}-H)^2}{2{\cdot}32(2\kappa^2-1)},$$

where G_N is the Gibbs' function of the normal state. The second term in the right-hand side is clearly the free energy of the flux-lattice in the superconductor. The total length of flux-line per unit volume is B/ϕ_0, and the Gibbs' function per unit length of flux-line, in the lattice, is :

$$g = \frac{-\mu_0\phi_0(H_{c2}-H)^2}{2{\cdot}32(2\kappa^2-1)B}. \tag{6}$$

This includes magnetic and inter-flux-line interaction terms as well as the core energy, and also sums the energy for each flux-line over a radius $d/2$. An approximation to the core energy alone may be arrived at by multiplying the Gibbs' function per unit volume by the volume of the flux-line core (Love 1970). Assuming a radius of ξ for the flux-line core :

$$g_c = \frac{-\pi\xi^2\mu_0(H_{c2}-H)^2}{2{\cdot}32(2\kappa^2-1)}. \tag{7}$$

When the pinning centres are large, that is $a > d$ (volume pins), a volume of the flux-lattice is accommodated within each pin, and ΔW is equal to the change in g as defined by eqn. (6). For small pinning centres, when $a < d$ (point or surface pins) then ΔW is equal to the change in the core energy, g_c, as defined by eqn. (7). It is obvious that, for non-superconducting pins, $\Delta g = g$ and $\Delta g_c = g_c$. The appropriate values for $\Delta\kappa$ pinning are obtained by differentiating eqns. (6) and (7) with respect to κ.

Expressions for ΔW for the various types of pinning, for $\kappa \gg 1$, are shown in the table.

3.2. *The interaction distance, x*

For magnetic pinning, the interaction distance is the distance over which the magnetic induction changes, and this is the penetration depth λ. In a flux-line lattice, the order parameter ψ varies sinusoidally with a wavelength of $2d$. The maximum force obtained by differentiating the sinusoidal expression is $\pi\Delta W/d$; i.e. $x = d/\pi$. For volume pins, where the full expression for the flux-line energy is appropriate, this is the correct value for x, though little is lost by, as is usual, simply assuming $x = d$. For point or surface pins, in which the dimension of the pins parallel to the direction of the Lorentz force, a is $< d$, then the energy of the core alone is appropriate. x should then be the size of the core, $\approx \xi$. Campbell and Evetts (1972) have pointed out that for a spherical pin, the energy changes continuously as the flux-line moves from the centre to the edge of the pin, and $x = a/2$, the radius of the pin.

3.3. *The length of interacting flux-line, L*

L is the total length of flux line per unit volume that is involved in the pinning interaction and depends upon whether the pins are points, surfaces or volumes. If the flux-lines are perfectly rigid, then each will, on average,

Type of interaction	Geometry of pin	L	x	Type of centre	ΔW	Pinning function, $f_p(h)$	Equation No.	Position of maximum
Magnetic	Volume	$\frac{S_v}{d}$	λ	Normal	$\frac{-\phi_0(H_{c2}-H)}{2{\cdot}32\kappa^2}$	$\frac{\mu_0 S_v H_{c2}^2 h^{1/2}(1-h)}{\kappa^3}$	8	$h=0{\cdot}33$
				$\Delta\kappa$	$\frac{-\phi_0(H_{c2}-2H)\Delta\kappa}{2{\cdot}32\kappa^3}$	$\frac{\mu_0 S_v H_{c2}^2 h^{1/2}(1-2h)\Delta\kappa}{\kappa^4}$	9	$h=0{\cdot}17, 1$
Core	Volume	$\frac{S_v}{d}$	d	Normal	$\frac{-\mu_0\phi_0(H_{c2}-H)^2}{4{\cdot}64\kappa^2 B}$	$\frac{\mu_0 S_v H_{c2}^2(1-h)^2}{5{\cdot}34\kappa^2}$	10	—
				$\Delta\kappa$	$\frac{-\phi_0(H_{c2}-H)\Delta\kappa}{2{\cdot}32\kappa^3}$	$\frac{\mu_0 S_v H_{c2}^2 h(1-h)\Delta\kappa}{2{\cdot}67\kappa^3}$	11	$h=0{\cdot}5$
	Surface	$\frac{S_v}{d}$	ξ	Normal	$\frac{-\pi\xi^2\mu_0(H_{c2}-H)^2}{4{\cdot}64\kappa^2}$	$\frac{\mu_0 S_v H_{c2}^2 h^{1/2}(1-h)^2}{4\kappa^2}$	12	$h=0{\cdot}2$
				$\Delta\kappa$	$\frac{-\pi\xi^2\mu_0 H(H_{c2}-H)\Delta\kappa}{2{\cdot}32\kappa^3}$	$\frac{\mu_0 S_v H_{c2}^2 h^{3/2}(1-h)\Delta\kappa}{2\kappa^3}$	13	$h=0{\cdot}6$
	Point	$\frac{BV_f}{\phi_0}$	$\frac{a}{2}$	Normal	$\frac{-\pi\xi^2\mu_0(H_{c2}-H)^2}{4{\cdot}64\kappa^2}$	$\frac{\mu_0 V_f H_{c2}^2 h(1-h)^2}{4{\cdot}64a\kappa^2}$	14	$h=0{\cdot}33$
				$\Delta\kappa$	$\frac{-\pi\xi^2\mu_0 H(H_{c2}-H)\Delta\kappa}{2{\cdot}32\kappa^3}$	$\frac{\mu_0 V_f H_{c2}^2 h^2(1-h)\Delta\kappa}{2{\cdot}32a\kappa^3}$	15	$h=0{\cdot}67$

have a fraction V_f of its length inside a pinning centre, where V_f, the volume fraction of the superconductor, $\approx (a/l)^3$. In practice flux-lines will have some flexibility and will curve so as to interact with more pins. A perfectly flexible flux-line would have a fraction (a/l) of its length within pinning centres. Actual values will lie between these two limits, normally nearer the former; the latter, however, applies when $d=l$, leading to matching effects (Peterman 1970).

The total length of flux-line per unit volume is B/ϕ_0 and for point pins, in the absence of matching effects, $L \approx V_f \; B/\phi_0$. Where the pinning is a surface interaction either at surface pins or at the interface with volume pins, only those flux-lines located at the interface are affected. In this case $L = S_v/d$ when S_v is the surface area per unit volume projected in the direction of the Lorentz force (Campbell *et al.* 1968). $S_v \approx a^2/l^3$ for large precipitates; for dislocation cell-structures and sub-grain boundaries $S_v \approx 1/l$ where l is now the cell diameter.

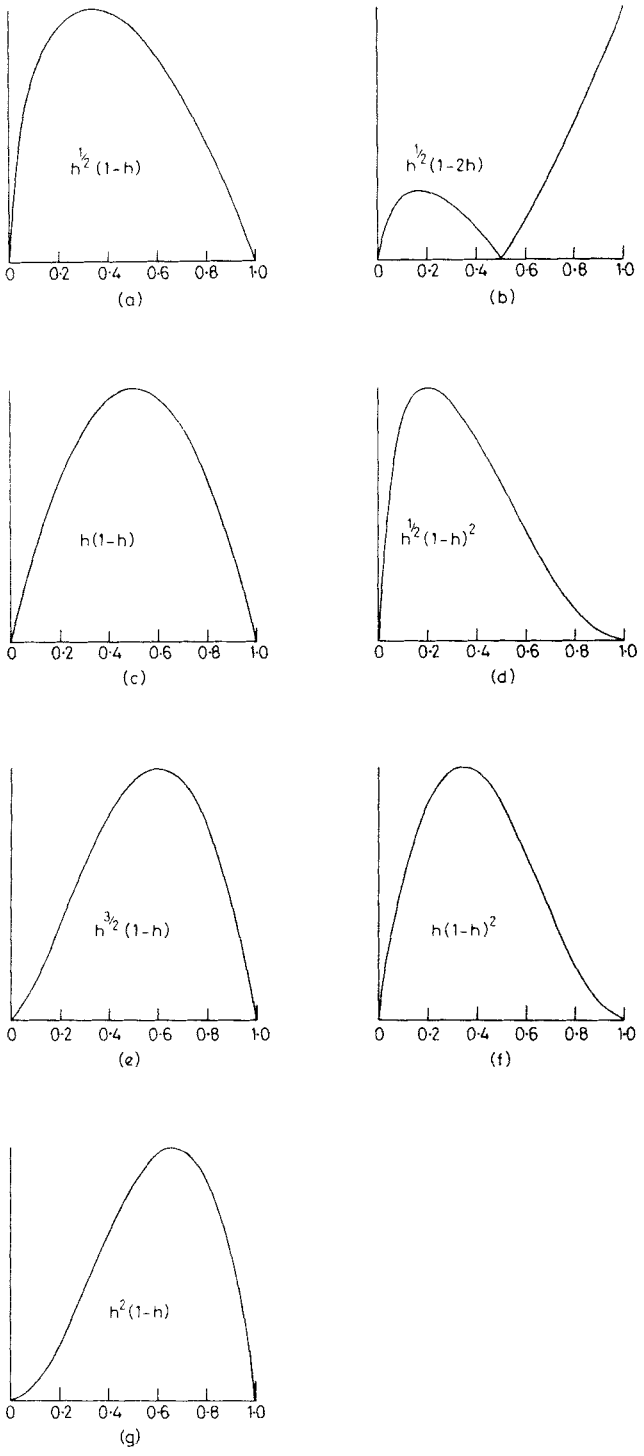
3.4. *The efficiency factor*, η

Account must be taken of flux-lattice elasticity whenever the inter-flux-line forces are stronger than the pinning forces, as would be the case for a dense array of individually weak pins. For strong pins where the reverse obtains, the flux-lattice may reasonable be ignored, as it is sufficiently disrupted in the neighbourhood of the pinning centre to allow flux-lines to take up positions of minimum energy (maximum pinning) without restraint. This appears to be the case for pinning by normal bismuth particles in a Pb-Bi eutectic (Coote, Evetts and Campbell 1972) and for pinning by dislocation tangles in Nb-Ti (Hampshire and Taylor 1972). This behaviour may be typical of most hard superconductors. Träuble and Essmann (1968) have shown, by the high-resolution Bitter technique, that at high current densities the flux-lattice breaks down and becomes amorphous. Herring (1973) has combined this technique with transmission electron microscopy to observe simultaneously the flux distribution and the underlying defect structure in thin films of niobium. The flux-lattice is shown to be distorted in conformity with the positions of crystal dislocations; when the niobium contains a cell-structure the flux-lattice becomes polycrystalline, with grain boundaries coinciding with the cell-walls in the niobium.

It is not clear how far these observations may be taken as indicative of behaviour within the bulk of a superconductor. Pearl (1966) has shown that the structure of, and forces between, flux-lines close to the surface of a superconductor differ from those in the interior. The above results may be typical only of the surface, but they are the only direct information currently available. The presence of dislocations in the flux-line lattice, as observed by Träuble and Essmann, is expected to allow local relaxation of the lattice in the neighbourhood of pinning centres (Coote 1970, Dew-Hughes 1971). It will be assumed in what follows that the structure of the flux-lattice and its elasticity can be ignored, and that the efficiency factor $\eta = 1$.

3.5. *Specific pinning functions* F_p

Pinning functions for specific situations, magnetic or core, normal or $\Delta\kappa$, point, surface or volume, may be determined by inserting the appropriate



$fn(h)$ versus h for the various expressions given in the table.

values of L, ΔW and x, as given in the preceding sections, into eqn. (3). Of all the possible combinations of the variables several are physically unrealistic. Magnetic pinning only occurs when a, l, $> \lambda$. Values of d range from $\approx \lambda$ at H_{c1} to $\approx \xi$ at H_{c2}. Thus for magnetic pinning a is always $> d$, and only volume pins can give rise to a magnetic interaction. The pinning functions for the remaining possibilities are summarized in the table. The functions are expressed in terms of the reduced field $h(=H/H_{c2})$; and appropriate substitutions are made for other quantities, e.g. $d=1{\cdot}07\ (\phi_0/B)^{1/2}$, $\lambda=\kappa\xi$, and $\xi=(\phi_0/2\pi\mu_0 H_{c2})^{1/2}$. The shapes of the various pinning functions are plotted in the figure.

Two of these results deserve further comment. The function for magnetic, $\Delta\kappa$, pinning (eqn. (9)), has a small peak at $h=0{\cdot}17$, goes to zero at $h=0{\cdot}5$, when it changes sign and should drop abruptly to zero again at $h=1$. The reversal of sign at $h=0{\cdot}5$ means that below this value of field the flux has a lower energy in the high-κ region, above this value of field its energy is lower in the low-κ region. Both regions have different values of H_{c2}, and superconductivity as measured by a resistanceless current will persist up to the upper critical field of whichever region is continuous. If this is the low-κ region, the Lorentz force and J_c will drop abruptly at H_{c2} for this region. This has been offered as a possible explanation for the 'peak-effect' in certain circumstances (Dew-Hughes 1971). When the continuous region has the higher κ, e.g. for a dislocation cell-structure (Dew-Hughes 1966), the Lorentz force goes through a second maximum at H_{c2} for the low-κ region, and then decreases gradually to zero at H_{c2} for the high-κ region (Dew-Hughes and Whitcomb 1972). This second peak will occur at $h=[1-(\Delta\kappa/\kappa)]$ where the reduced field h is calculated with respect to the higher of the two H_{c2} values, i.e. the experimentally determined H_{c2}.

The function for core, normal, volume pinning, (eqn. (10)) does not have a maximum, but decreases monotonically from its value at $h=0$ to zero at $h=1$. However, for volume pinning at low fields $a > d \approx \lambda$, and the magnetic interaction is appropriate. Core, volume pinning is only expected to operate at high fields as d decreases, and a pinning function described by eqns. (12) or (14) at low fields may follow eqn. (10) at high fields.

When the 'wave-length' of the pinning microstructure is $\approx \lambda$, both magnetic and core interactions may be appropriate. A comparison of the relative strengths of the two interactions may be made by looking at the ratios of eqn. (8) to eqn. (10), and eqn. (9) to eqn. (11). From this it can be seen that in materials of low-κ value (< 5 for normal pinning, $< 2{\cdot}5$ for $\Delta\kappa$ pinning), the magnetic interaction is the stronger of the two, and is likely to be the dominant pinning mechanism. In high-κ materials, the core interaction is stronger, the more so the higher the value of κ. In high-κ materials with a mixed microstructure, fine pins with a core interaction will be more effective than coarse, magnetic pins, though the latter may provide a greater interfacial area.

It can be seen from the table that this approach to mechanism of flux-pinning can predict divers pinning functions, depending upon the assumed nature of the pinning process. In the next section these predictions are compared with experimental determinations of Lorentz force curves in a variety of materials.

§ 4. Comparison with experiment

Comparison of predicted pinning functions with experiment requires careful measurement of J_c versus H over as wide a temperature range as possible, together with an accurate determination of H_{c2} at each temperature. H_{c2} is usually estimated by an extrapolation of J_c versus H to $J_c = 0$. It is common to make a linear extrapolation, though some authors claim that it is more appropriate to extrapolate $J_c^{1/2}$ versus H (Montgomery and Sampson 1965, Shapira and Neuringer 1965). The choice of extrapolation procedure will determine the ascertained value of H_{c2}. More serious is the possibility that H_{c2} may be paramagnetically limited. It is obvious from the way in which the pinning functions are derived that the value of H_{c2} which should be used is the non-paramagnetically limited, Abrikosov value. Incorrect values of H_{c2} will affect the shape of the pinning function.

It must also be borne in mind that more than one pinning mechanism may be operative in a material. Their effects will add, and produce a new pinning function. For example, precipitation may be nucleated at the nodes of a pre-existing dislocation structure, or alternatively subsequent cold-work may establish a cell-structure whose nodes coincide with previously formed precipitates. This could give rise to a mixture of point, normal (eqn. (14)) and surface, (eqn. (13)) pinning, with a broad peak extending from $h \approx 0{\cdot}33$ to $h \approx 0{\cdot}6$. It is therefore only in very clearly defined cases that a true test of the theoretical predictions can be made.

The temperature dependence of the pinning functions given in the table is, in all cases, included in the term $(H_{c2})^2$, modified slightly by the temperature variation (relatively small) of κ. In practice the temperature dependence is found to be $(H_{c2})^n$, with usually $2 \leqslant n \leqslant 3$; the lower value being the one most commonly found. Thus the functions given in the table may account for the majority of cases, but the possibility of their being other pinning mechanisms, with values of $n > 2$, must not be ruled out. It should be noted, however, that if x is assumed equal to ξ, instead of $a/2$, for core, point pinning, $n = 2{\cdot}5$ though $fn(h)$ remains as in eqns. (14) and (15) (the constants are also changed slightly).

Magnetic, normal pinning, as described by eqn. (8), has been verified in lead-bismuth eutectic alloys (Campbell *et al.* 1968, Coote *et al.* 1972) and by a variety of normal particles in a range of lead-bismuth solid solutions (Bibby 1970). The proportionality of F_p to S_v is also established in these systems. Very little doubt can be cast on the validity of this mechanism for the systems in which it is appropriate.

Qualitative verification of magnetic, $\Delta\kappa$ pinning, eqn. (9), by dislocation cell-structures has been obtained in Mo-34% Re (Dew-Hughes and Witcomb 1972) and in deformed and lightly annealed Nb-65% Ti (Witcomb and Dew-Hughes 1973). Kroeger (1969) has found a peak at $h = 0{\cdot}85$ in Nb-5% Ti alloy, which may result from this mechanism if $\Delta\kappa/\kappa = 0{\cdot}15$, though the limited temperature-dependence data suggest that n is nearer 3 than 2.

Core, $\Delta\kappa$, volume pinning, eqn. (11), was originally proposed to explain results on heavily cold-worked Nb-60% Ti commercial material (Hampshire and Taylor 1972). Its validity for this material has recently been extended over the whole field range from H_{c1} to H_{c2} (Hampshire, private communication). It has also been proposed to explain pinning by titanium sub-oxides in heat-treated Nb-65% Ti (Witcomb and Dew-Hughes 1973).

Data on Nb-25% Zr (Aron and Ahlgren 1968) as analysed by Kramer (1973), clearly fit a function $h^{3/2}(1-h)$, with $n=2$. This is in agreement with core, $\Delta\kappa$, surface pinning, eqn. (13), as might be expected from pinning by a fine dislocation network. The results of Fietz and Webb (1969) in various dilute niobium alloys, also fit this form of $fn(h)$, over most of the field range, but in this case $n=2{\cdot}5$.

Kramer's (1973) analysis of Haller and Belanger's (1971) data on Nb_3Sn tape shows clearly that the appropriate pinning functions follows $h(1-h)^2$, again with $n=2$. This suggests that eqn. (14), core, normal, point pinning is appropriate. Diffused Nb_3Sn tape doped with ZrO_2 particles (Benz 1968) follows the same function, as does Enstrom and Appert's (1972) data on some of their vapour-deposited Nb_3Sn doped with various gases. Other samples of their material, with values of critical Lorentz forces lower by a factor $\sim 0{\cdot}1$, show a more complicated dependence upon h, with a shoulder at $h\sim 0{\cdot}33$, and a peak at $h=0{\cdot}6$. This is suggestive of core, normal point pinning, eqn. (14), plus core, $\Delta\kappa$, surface pinning, eqn. (13). These materials contain a dispersion of small, non-superconducting precipitates, whose presence during the growth process results in a fine-grained structure of Nb_3Sn. A mixture of normal, point pins (the precipitates), and $\Delta\kappa$, surface pins (the grain boundaries), is therefore expected. Comparison of eqns. (13) and (14), for $V_f/a=S_v$, shows, assuming $\Delta\kappa/\kappa\sim 0{\cdot}1$, that the normal pinning force will be some five times stronger than the $\Delta\kappa$ pinning force. Precipitate pinning will therefore predominate, with a peak at $h=0{\cdot}33$, except in the case of some of Enstrom and Appert's weakly pinning material, where presumably the amount of precipitation is limited, and the grain-boundary, $\Delta\kappa$, pinning predominates with a peak at $h=0{\cdot}6$.

V_3Ga tape manufactured by a diffusion-reaction route is a material with interesting commercial possibilities, because its critical current density at high fields ($\geqslant 14T$) is higher than that for comparable Nb_3Sn tape. There is not sufficient published data to determine the exact form of the pinning function for this material, but from manufacturers' data (Vacuum Metallurgical Co. Ltd, Japan) and that of Tachikawa and Iwasa (1970), a peak in F_p occurs at $h\approx 0{\cdot}6$ or $\approx 0{\cdot}67$. This is suggestive of eqns. (13) or (15), i.e. core, $\Delta\kappa$, surface or core, $\Delta\kappa$, point pinning. A possible explanation due to the author (Dew-Hughes 1974) is enlarged upon here.

Samples of many of the superconducting A15 compounds are known to undergo a martensitic transformation at low temperatures, to a tetragonal structure. Suppression of the transformation by pressure in V_3Si reduces flux-pinning (Brand and Webb 1969), and the suggestion that it may be a cause of flux-pinning is reasonable. Livingston (1966) has shown that in In-Tl alloys, maximum flux-pinning is observed when only about half of the alloy has undergone a transformation to martensite. Comparison may be made with magnetically hard steels, in which domain boundaries are pinned by retained austenite. It is postulated that a partial transformation will produce a variation in κ; either the transformed regions have a value of κ which is intrinsically different from that of the parent material, or the closely-spaced twin-boundaries resulting from the secondary shear that is invariably associated with a martensite transformation (Bilby and Christian 1961) will, via an added contribution to the normal state resistivity, lead to

an increase in κ for the transformed regions. The appropriate pinning function will depend upon the geometry of the transformed regions and eqns. (11), (13) or (15), with peaks at $h = 0{\cdot}5$, $0{\cdot}6$ or $0{\cdot}67$, are to be expected. Partial transformation in V_3Ga is known to be encouraged by fine grain-size (Nembach, Tachikawa and Takano 1970), and this may explain the rather curious dependence of J_c upon grain-size observed by Nembach and Tachikawa (1969). The existence of a finer grain-size and/or the presence of precipitates may suppress the martensite transformation in commercial Nb_3Sn.

§ 5. Conclusions

It is concluded that this approach to the problem of flux-pinning, which ignores flux-lattice elasticity effects, is able to predict many forms of the pinning function and of the observed scaling laws, and offers a reasonable explanation for the mechanisms of flux-pinning in many model systems and commercial superconducting alloys. More data, particularly on commercial Nb_3Sn and V_3Ga, are needed to give a rigorous test of the ideas presented here. In particular H_{c2} must be carefully defined. Confusion can also arise when more than one pinning mechanism is in operation.

It is natural that this theory will be compared to those based on flux-lattice elasticity. It is here claimed that a better agreement with a wider range of experimental results is achieved for this theory than for the original elasticity theory of Labusch (1969) or for any subsequent modification thereof.

The most recent attempt to provide an explanation of the observed scaling laws is due to Kramer (1973). Briefly, Kramer's theory involves two competing processes. One is the model of Labusch (1969) and Fietz and Webq (1969) in which elastic relaxation of the flux-line lattice allows of pinning. The flux-lattice elastic constants decrease as B increases allowing greater relaxation and stronger pinning. At high fields it is proposed that flux-lines do not become unpinned, but that the flux-line lattice undergoes shear. The resistance of the lattice to shear decreases as B increases. The maximum in $F_p(h)$ occurs when pinning gives way to shear. The latter process is assumed to be independent of the pinning strength, which, as it increases, causes the peak to move to lower fields. The theory is elegant, and academically satisfying as it purports to be applicable, by an appropriate adjustment of parameters, to all materials.

There are, however, several possible objections to this theory. It is assumed, as do all pinning theories to date based on flux-lattice elasticity, that pinning centres are points or lines. There is ample experimental evidence in favour of pinning by interfaces and bulk defects in a majority of materials (Dew-Hughes 1971). Other evidence which belies the necessity for involving elasticity effects is quoted above (§ 3.4). It is difficult to imagine any element of a three-dimensional flux-lattice, randomly pinned, being able to undergo shear without somewhere having to be first unpinned. Contrary to many experimental observations, this theory predicts that the Lorentz force always has positive curvature close to H_{c2}. Whilst it is true that in many cases treatments which increase the pinning strength also shift the peak to lower values of the reduced induction, this is by no means universally true, and there are examples where the peak moves to higher values of b (Coote 1970).

The one area in which lattice elasticity has had any success is in explaining the ' peak effect ' (Pippard 1969, Campbell and Evetts 1972, van der Klein, Kes and de Klerk 1974). It was postulated in § 3.4 that strong local pinning forces will disrupt the lattice ; elasticity effects are only likely to be observed for a fairly dense distribution of individually weak pins. In general materials which show a peak effect have rather low current densities ; treatments which increase the current density cause the disappearance of the peak. The disregard of lattice elasticity in applying this theory to strong pinning, high critical current materials is justified by its success.

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