



Extended analysis of gravitomagnetic fields in rotating superconductors and superfluids

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Abstract

Applying the Ginzburg–Landau theory including frame dragging effects to the case of a rotating superconductor, we were able to express the absolute value of the gravitomagnetic field involved to explain the Cooper pair mass anomaly previously reported by Tate. Although our analysis predicts large gravitomagnetic fields originating from superconductive gyroscopes, those should not affect the measurement of the Earth gravitomagnetic field by the Gravity Probe-B satellite. However, the hypothesis might be well suited to explain a mechanical momentum exchange phenomena reported for superfluid helium and a dragging force present in a rotating superconductor experiment.

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1. Introduction

The authors recently published a paper [1], suggesting, that a reported disagreement between

experimental measurements and theoretical predictions for the Cooper pairs mass [2,3] might arise from a non-classical gravitomagnetic contribution (also known as frame dragging or Lense-Thirring effect). In normal matter, the ratio between electromagnetic and gravitational fields is given by the difference in the respective permeabilities [4]. However, magnetic fields generated by rotating superconductors as a consequence of the quantization of the canonical momentum of the Cooper pairs

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do not depend on the permeability. Hence, there is the possibility that the ratio between those two fields might be different in a superconductor.

Tate et al. [2,3] used a SQUID to measure the magnetic field generated by a thin niobium superconductor, also called the London moment. This experiment was done as a preparation activity for the Gravity-Probe B satellite. As the thickness of the ring was on the order of the London penetration depth, the magnetic field will be zero at certain frequency steps defined by the quantization number n according to the Ginzburg–Landau equation. This zero-flux condition was then used to derive \hbar/m . In our previous paper, [1] we added a gravitomagnetic component ΔB_g to this frequency step $\Delta\nu$ in the zero-flux condition to account for the deviation of the measured Cooper-pair mass compared to its theoretical value. This gravitomagnetic contribution was found to be $\Delta B_g = 1.65 \times 10^{-5} \text{ rad s}^{-1}$ (value corrected² from Ref. [1]), where $m^* = 1.82203 \times 10^{-30} \text{ kg}$ is the Cooper pair mass measured by Tate, $m = 1.82186 \times 10^{-30} \text{ kg}$ is the theoretical mass of the cooper pairs including relativistic corrections, $S = 2.02351 \times 10^{-3} \text{ m}^2$ is the area of the superconductive ring used by Tate, and h is the Planck constant.

Besides our previous paper [1], several other approaches have been described in the recent literature to explain Tate’s Cooper pair mass anomaly, however, all of them have not been able to solve the anomaly (in fact, the corrections proposed further reduce the theoretical Cooper pair mass whereas the one measured by Tate is larger than two times the electron mass) [5–7]. In this paper, we will extend our argument and express the absolute value of the gravitomagnetic field possibly involved in rotating superconductors to account for the theoretical disagreement. We also add experimental evidence for superfluids supporting our conjecture, and we will analyse possible consequences on the results to be expected from the Gravity-Probe B satellite.

2. Gravitomagnetic fields in superconductors

Tate et al. [2,3] followed Ginzburg–Landau theory, integrating the current density of Cooper pairs around a closed path including the effect of a rotating reference frame, but neglecting any gravitomagnetic fields, since the densities and speeds involved are very small from a relativistic point of view,

$$\frac{m^*}{e^2 n_s} \oint_{\Gamma} \vec{j} \cdot d\vec{l} = \frac{nh}{2e} - \int_{S_r} \vec{B} \cdot d\vec{S} - \frac{2m^*}{e} \cdot \vec{\omega} \cdot \vec{S}_r, \quad (1)$$

where n_s is the Cooper–electron number density, S_r the area bounded by the closed line Γ inside the superconductor, ω the angular velocity, B the London moment and m^* the Cooper-pair mass measured by Tate. The argument put forward in Ref. [1] says that Tate’s measured Cooper-pair mass $m^* = 1.82203 \times 10^{-30} \text{ kg}$ is higher than the theoretically expected one $m = 1.82186 \times 10^{-30} \text{ kg}$ because Eq. (1) does not account for a possibly involved gravitomagnetic field B_g . The complete Ginzburg–Landau equation should then read as

$$\begin{aligned} \frac{m}{e^2 n_s} \oint_{\Gamma} \vec{j} \cdot d\vec{l} = & \frac{nh}{2e} - \int_{S_r} \vec{B} \cdot d\vec{S} - \frac{m}{e} \int_{S_r} \vec{B}_g \cdot d\vec{S} \\ & - \frac{2m}{e} \cdot \vec{\omega} \cdot \vec{S}_r. \end{aligned} \quad (2)$$

Combining both equations we can express the gravitomagnetic field B_g as

$$\vec{B}_g = 2\vec{\omega} \cdot \left(\frac{m^* - m}{m} \right) + \left(\frac{m^* - m}{m} \right) \cdot \frac{1}{S_r n_s} \oint_{\Gamma} \vec{j} \cdot d\vec{l}. \quad (3)$$

In a superconductor that is thick compared to the London penetration depth, the current integral in the above equations can be set to zero as there is always a path inside the superconductor in which no current is flowing (outside the London penetration depth). The gravitomagnetic field is then just a function of the angular velocity and of the mass difference of the Cooper-pairs

$$\vec{B}_g = 2\vec{\omega} \cdot \left(\frac{m^* - m}{m} \right) = 2\vec{\omega} \frac{\Delta m}{m}. \quad (4)$$

² The corrected equation is, $\frac{\hbar}{m} = 2S \left(\Delta\nu + \frac{\Delta B_g}{4\pi} \right)$, together with $\frac{\hbar}{m^*} = 2S\Delta\nu$ one can compute ΔB_g .

According to Tate's results, the gravitomagnetic field in her configuration but using a thick rotating niobium superconductor would then be

$$\vec{B}_g = \vec{\omega} \cdot 1.84 \times 10^{-4}, \quad (5)$$

which is very large even for small angular velocities compared to the gravitomagnetic field produced by the Earth (about 10^{-14} rad s⁻¹), but cannot be ruled out based on the experimental results achieved so far. It provides a mean to actually solve Tate's mass anomaly, whereas all other theoretical approaches in literature [5–7] would even increase the anomaly.

NASA and Stanford University launched the Gravity-Probe B (GP-B) satellite in April 2004 to measure the gravitomagnetic field (Lense-Thirring effect) of the Earth [8] by precession of superconductive gyroscopes relative to a fixed star. The gyroscope consists of a fused quartz ball covered with a 1.5 μ m thick niobium layer. This ball is rotated at a temperature of 2 K and therefore produces a magnetic field being a rotating superconductor. The change of axis orientation is measured by SQUID magnetometers. According to general relativity, the geodetic precession rate is 6.6 arc-s/yr and the Lense-Thirring precession rate is 0.042 arc-s/yr. Since these gyroscopes are rotating superconductors with a niobium thickness larger than the London penetration depth (about 85 nm), a gravitomagnetic field according to Eq. (5) should be generated if our conjecture is correct. Would that influence the measurements on GP-B?

The spin angular momentum of GP-B's superconductive gyroscopes, measured by a co-moving observer, \vec{S}_0 , obeys the equation

$$\frac{d\vec{S}_0}{dt} = \vec{\Omega} \times \vec{S}_0, \quad (6)$$

where $\vec{\Omega}$ includes the Thomas-precession, the geodetic and the gravitomagnetic (Lense and Thirring frame dragging) precessions [9].

Our conjecture leads to a modification of the superconductive gyroscope's spin angular momentum according to:

$$\vec{S}_0 = I_{SC}\omega_{SC} \left(1 + 2 \frac{\Delta m}{m} \right), \quad (7)$$

where I_{SC} and ω_{SC} are the moment of inertia and the angular velocity of the superconductive gyroscopes respectively. The product $S_{c0} = I_{SC}\omega_{SC}$ is the classical angular momentum measured in the co-moving frame. Doing (7) into (6) leads to:

$$\frac{d\vec{S}_{c0}}{dt} = \vec{\Omega} \times \vec{S}_{c0}. \quad (8)$$

The quantum contribution to the angular momentum cancels out, leaving the usual relativistic equation, ruling the behaviour of GP-B's gyroscopes, unchanged. Therefore the anomalous gravitomagnetic moment predicted by our conjecture will have no influence on the geodetic and gravitomagnetic precessions measured in the GP-B experiment!

3. Gravitomagnetic fields in superfluids

In our previous paper [1], we discussed the quantization of the full canonical momentum including a gravitomagnetic vector potential A_g for the case of a neutral superfluid:

$$\oint \vec{p}_s \cdot d\vec{l} = \oint (m\vec{v}_s + m\vec{A}_g) \cdot d\vec{l} = nh. \quad (9)$$

As there is no comparable situation like a thick superconductor where n can be set to zero, the gravitomagnetic field generated by a superfluid can not be expressed easily as for thick superconductors in Eq. (4). However, according to the Feynman–Onsager-vortex model, the first vortex in a superfluid ($n = 1$) only appears if the angular velocity is larger than a critical angular velocity [10] ω_c

$$\omega_c = \frac{\hbar}{mR^2} \ln \left(\frac{R}{a} \right), \quad (10)$$

where R is the inner radius of the superfluid cylindrical container and a is the effective radius of the vortex core. In the case of superfluid helium-I, $a \cong 10^{-10}$ m. For a container's rotational speed below ω_c the superfluid will be in a non-rotational state, which is also called the rotational Meissner effect. This has been experimentally confirmed by Hess and Fairbank [10]. In this specific case, we can write Eq. (9) as

$$\vec{B}_g = -2\vec{\omega} \quad \omega \leq \omega_c. \quad (11)$$

But due to the non-rotational state of the superfluid, the angular velocity of the superfluid will be zero and hence $B_g = 0$. Let us now switch reference frames between the superfluid bulk and the container, this is equivalent to the case that the container is at rest and the superfluid is rotating. Here Eq. (11) should apply and an equivalent gravitomagnetic field produced accordingly. For that case, Eq. (11) would correspond exactly to the gravitational analogue of the magnetic Barnett effect, that we formerly called the “Gravitomagnetic Barnett Effect” [11].

For angular velocities of the container greater than ω_c , a gravitomagnetic field could be generated, modulated by a function similar to Eq. (3) in the case of thin superconductors, where we can write

$$\vec{B}_g = \frac{n}{S} \frac{h}{m} - 2\vec{\omega} \quad \omega > \omega_c, \quad (12)$$

where n is the total number of vortices present in the superfluid and S is the integration area. The conservation of angular momentum in rotating superfluids would therefore require the generation of a gravitomagnetic field through the cross section of the rotating container. In the case of multiply-connected containers, this field should also be present in the volume that does not contain the superfluid; similarly as for the magnetic case with rotating superconductors.

Due to accelerations and decelerations of the container, the changing gravitomagnetic fields will generate so-called gravitoelectric (also called non-Newtonian gravitational) fields [4], which would then act on the superfluid, as it has mass:

$$\text{rot } \vec{g} = -\frac{d\vec{B}_g}{dt}. \quad (13)$$

In fact this seems to be a logical way to add momentum to the fluid in its superfluid state as there is no friction and no charge (besides using large magnetic fields that act on the small magnetic moments of the helium atoms). Due to the double minus signs in Eqs. (11) and (13), the gravitational field would act in the same direction as ω .

This behaviour was actually measured in a follow up paper from Hess after their detection of the rotational Meissner effect in superfluids [12]. He mounted the superfluid helium container on an extremely low friction magnetic bearing to study the momentum exchange of the superfluid with the container. After cooling the helium at rest below its critical temperature, he rotated the container and observed that nearly all momentum that he injected in the container was transferred to the superfluid (vortices appeared)—although a superfluid has no friction. Also the other direction worked in a similar way: the liquid was first cooled during rotation below its critical temperature. Then the angular momentum of the container was reduced and, accordingly, also the superfluid was found to lose vortices and momentum. Hess called it mechanical generation/deletion of vortices in superfluids—without an apparent explanation. According to the knowledge of the authors, no subsequent experiments of a similar kind have been carried out to further characterize this phenomenon in superfluids.

The superfluid gravitational behaviour is opposite to that of a superconductor, as in this case, the gravitomagnetic field points in the same direction as the angular velocity (Eq. (5)). Therefore, combining Eqs. (5) and (13), the gravitational field generated by an angularly accelerating superconductor will appear in opposite sign to the acceleration and hence will act as a dragging force which slows the superconductor down.

4. Conclusions

We extended the analysis of our previous paper [1] to show how large gravitomagnetic fields could be involved in rotating superconductors and superfluids. An absolute value of the gravitomagnetic field necessary to explain Tate’s Cooper pair mass anomaly was derived Eq. (4). Although being a large effect compared to the gravitomagnetic field generated by the Earth itself, the influence on the measurements to be performed by the Gravity-Probe B satellite appears to be irrelevant.

After further analysing the case of rotating superfluids, we found also support for our conjecture being able to explain the mechanical generation/deletion of vortices in superfluid helium by rotation of the superfluid's container. This is an intriguing simple argument to explain frictionless momentum exchange between the superfluid and the container.

These results shall stimulate experimental and theoretical work to prove or disprove the conjectured large gravitomagnetic fields postulated. According to our analysis, the fields should be large enough to be detected in a laboratory setup investigating non-stationary rotational regimes. Presently, an experiment is underway at ARC Seibersdorf research to test the conjecture outlined in this paper.

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