in which case the rotation is affected but not the ellipticity. While this approximation appears satisfactory for sample A, it is highly probable that a corresponding treatment for high conductivity specimens would produce a marked effect on Δ as well as on θ, particularly in the region of a change of sign.

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ONSET OF SUPERCONDUCTIVITY IN DECREASING FIELDS

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We show that in ideal samples the nucleation of superconducting regions in decreasing fields should always occur near the surface of the sample. As a result the nucleation field is not equal to the Landau value [1] $H_{c2} = \varkappa\sqrt{2}\, H_c$ (where $\varkappa$ is the Landau-Ginzburg parameter) but is given by $H_{c3} = 2.392\, \varkappa H_c$. For a superconductor of the first kind, this implies that the values of $\varkappa$ derived from supercooling experiments must be corrected. For a superconductor of the second kind, the conclusion is that in fields H between H_{c2} and H_{c3} there remains a superconducting sheath on some parts of the sample.

We derive the nucleation field in the Landau-Ginzburg region. The linearised equation to be solved is (in the conventional notation [1])

$$\frac{1}{2m}\left[-\mathrm{i}\hbar\nabla - 2e\frac{A}{c}\right]^2 \psi + \alpha\psi = 0 \,, \qquad (1)$$

where $H = \operatorname{curl} A$ is simply the applied field. In connection with eq. (1) it is often convenient to introduce the characteristic length $\xi(T)$ defined by

$$\xi^2(T) = -\frac{\hbar^2}{2m\alpha} \,. \qquad (2)$$

We assume that a) $\xi(T)$ is small compared to the bulk dimensions of the sample [2], b) the surface polish is such that the local radii of curvature of the boundary are large compared with $\xi(T)$. This enables us to consider only the problem of a plane boundary. It will turn out that the favourable situation for nucleation occurs when the field H is parallel to the surface. Thus we take the boundary plane as yOz, the field being in the z direction and the metal occupying the half space $X > 0$. The half-space $X < 0$ is assumed to be a vacuum or an insulator. In this case the boundary condition to be applied to eq. (1) is, to a good approximation

$$\left[\left(-\mathrm{i}\hbar\frac{\partial}{\partial x} - \frac{2eA_x}{c}\right)\psi\right]_{x=0} = 0 \,. \qquad (3)$$

We choose the gauge $A_X = A_Z = 0$, $A_Y = HX$ and look for solution of the form *

$$\psi = f(x)\, \mathrm{e}^{\mathrm{i}ky} \,. \qquad (4)$$

Eqs. (1) and (3) become:

$$-\frac{\hbar^2}{2m}\frac{\mathrm{d}^2 f}{\mathrm{d}x^2} + \frac{1}{2m}\left[\hbar k - \frac{2e}{c}HX\right]^2 f = -\alpha f \,, \qquad (5)$$

* Taking $\psi = \mathrm{e}^{\mathrm{i}k'z} f(x,y)$ give an extra k'^2 contribution to the value of $-\alpha$. As we are interested in the lowest value of $-\alpha$ we take $k' = 0$.

$$\mathrm{d}f/\mathrm{d}x = 0, \qquad x = 0, \qquad x = \infty . \tag{6}$$

Eq. (5) is of the form of the Schrödinger equation for a harmonic oscillator of frequency $\omega = 2eH/mc$, the minimum of the potential being located at:

$$X_0 = \frac{\hbar ck}{2eH} . \tag{7}$$

The wave function is concentrated in region of dimension $\sim \xi(T)$ around X_0. When $X_0 \gg \xi(T)$ the boundary condition (6) is unimportant and we find the usual harmonic oscillator solution:

$$f = \exp[-\tfrac{1}{2}(\frac{X-X_0}{\xi(T)})^2] , \tag{8}$$

$$-\alpha = \tfrac{1}{2}\hbar\omega = \frac{e\hbar H}{mc} . \tag{9}$$

In the opposite limit $X_0 = 0$, the solution (8) again applies, since it satisfies (6) exactly; thus for $X_0 = 0$ we still find the eigenvalue (9).

We shall now show that for intermediate values of X_0 the eigenvalue is lowered below the value (9). We replace eq. (5) (applying for $x > 0$) and the boundary condition (6) by another Schrödinger equation, applying for $-\infty < x < \infty$, where the potential $V(X)$ is symmetrised:

$$\begin{aligned} V(X) &= \frac{2eH^2}{mc^2}(X-X_0)^2 \qquad (X > 0) ; \\ V(X) &= V(-X) \qquad (X < 0) . \end{aligned} \tag{10}$$

The lowest eigenvalue for the potential V has no nodes, is even, and thus satisfies automatically to condition (6).

For $X_0 > 0$, $V(X) < (2e^2H^2/mc^2)(X-X_0)^2$ in all region $X < 0$. Thus the lowest eigenvalue associated with $V(X)$ is smaller than the eigenvalue (9) associated with the potential $(2e^2H^2/mc^2)(X-X_0)^2$. This means that *nucleation will be easier in the vicinity of the surface*.

The next step is to find the value of X_0 for which the eigenvalue is a minimum. Using the variational principle on the free energy, this minimum condition can be written as:

$$\int f^2(\hbar k - \frac{2eH}{c}x)\mathrm{d}x = 0. \tag{11}$$

Eq. (11) shows that in the optimum state the over-all surface current vanishes. A detailed solution of (5) and (6) in terms of Weber functions shows that the optimum value of X_0 is $X_0 = \mu^2\xi(T)$ the corresponding eigenvalue being:

$$-\alpha = \mu^2 \frac{e\hbar H}{mc} \tag{12}$$

with $\mu^2 = 0.59010$. The coefficient μ is defined exactly by the implicit equation

$$\int_0^\infty \mathrm{d}t(2t-\mu)\, t^{-\frac{1}{2}+\frac{1}{2}\mu^2}\, \mathrm{e}^{-(t-\mu)^2} = 0 . \tag{13}$$

Eq. (12) corresponds to a field $H_{c3} = (1/\mu^2)\sqrt{2}\varkappa H_c$. This completes our discussion when the field is in the plane of the surface.

On the other hand, when H is normal to the surface, the Landau result (9) remains valid. We have not yet performed the calculation for intermediate angles, but according to all indications the field will vary smoothly with angle between H_{c2} and H_{c3}. The physical conclusions are:

1. Nucleation in an ideal specimen will always occur at the field H_{c3} (higher than the Landau value $H_{c2} = \varkappa\sqrt{2}H_c$) except if some very special steps are taken to avoid surface effects (e.g., temperature gradients or field inhomogeneities).

2. For superconductors of the first kind, starting from the experimental values of Faber [3] (reviewed by Lynton [1]) of the supercooling field we arrive at the following revised value of $\varkappa$: 0.0153(Al), 0.066(In), 0.0968(Sn). The theoretical values derived from $\varkappa = 0.96\ (\lambda_L(0)/\xi_0)$ (where $\lambda_L(0)$ and ξ_0 are taken from specific heat and anomalous skin effect measurements) are [1,4]: 0.01(Al), 0.051(In), 0.149(Sn).

3. For superconductors of the second kind, in fields H such that $H_{c2} < H < H_{c3}$ there will be a superconducting sheath near the surface of the sample. If the sample is a long cylinder with H along the axis, the sheath will cover all the surface of the cylinder. If it is a sphere, the sheath will be restricted to a small band near the equatorial plane when $H \sim H_{c3}$, but if H is decreased toward H_{c2} the sheath will progressively extend up to the poles.

These effects may explain some apparent discrepancies which occur between magnetic flux measurement of the transition field (determining H_{c2}) and resistivity measurements (determining more or less H_{c3} in simple geometries such as the cylinder described above [5]).

Of course, in non-ideal samples the volume defects can also participate in the nucleation process.

We end up with two remarks

a) The above calculation is valid only when the superconducting material is surrounded by an insulator. If the surface was coated by a normal metal, the boundary condition to be imposed on the Landau-Ginzburg wave function is strongly different from (3) and the nucleation field is modified. Calculations are in progress to investigate this situation.

b) A very different situation, where $\xi(T)$ is comparable to the sample dimensions, has been achieved in a recent experiment by Parks [7] with a thin film in a perpendicular field. The threshold

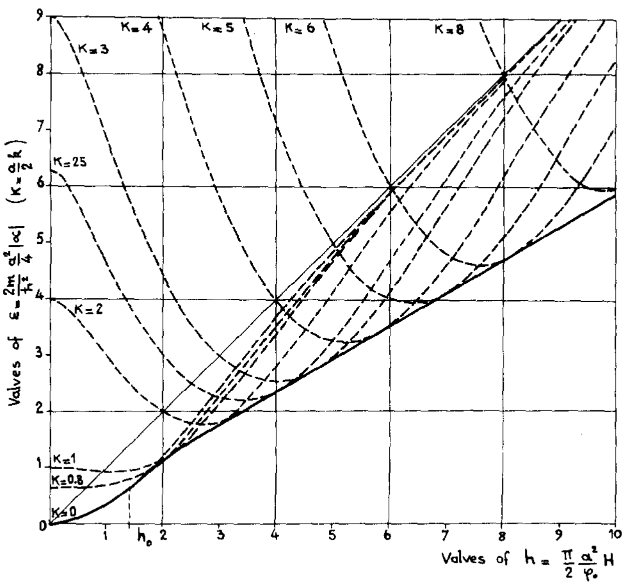

Fig. 1. Variation of the nucleation field in a slab.

field can again be derived from eq. (5), the boundary conditions (6) being modified to:

$$(\mathrm{d}f/\mathrm{d}x) = 0 \text{ for } x = \pm\tfrac{1}{2}a$$

(a is the sample thickness, i.e., in Park's experiment the cylinder diameter). We have also calculated this case. For $H < H_0$ ($H_0 \sim 2.75\ \varphi_0/\pi a^2$) (where $\varphi_0 = ch/2e$) we find that the optimum X_0 is 0: nucleation starts symmetrically. For $H > H_0$, X_0 moves toward the boundaries and progressively we recover the one boundary situation described above. The theoretical field is given on fig. 1. In practice Parks observes some anomalies at $H \sim 6.4\varphi_0/\pi a^2$ but a detailed interpretation of his results should take into account the (unknown) dissipative mechanisms which are responsible for the finite width of the resistive transition.

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ANGULAR DEPENDENCE OF THE VELOCITY OF SOUND IN SEMIMETALS

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Recently Harrison [1] has predicted a magnetic field dependence of the velocity of sound in metals. This dependence has been found experimentally in metals [2] and also in semimetals [3]. Harrison's theory is valid when the magnetic field is transverse to the direction of propagation of the sound wave. Rodriquez and Quinn [4] have also treated the case when the magnetic field is along the direction of propagation. The purpose of this note is to point out that interesting information about the Fermi velocity can be obtained from the dependence of the sound velocity s on the angle between the wave vector $\boldsymbol{q}$ of the sound wave and the magnetic field $\boldsymbol{H}$. The effect is similar to that found in the ultrasonic attenuation of semimetals [5,6].

In a paper submitted for publication elsewhere [7],