Appendix: Parameterising variable-strain critical-current data of strands for coils and magnets (Part of University of Durham Report No. EFDA/03-1103)

The "simplified interpolative scaling law" uses the following set of equations to parameterise the critical current density of technological $A15$ (Nb_3Sn and Nb_3Al) strands as a function of magnetic field, temperature, and axial strain (Ref. A1):

$$J_\mathrm{C}\left(B,T,\varepsilon_\mathrm{I}\right)=A\left(\varepsilon_\mathrm{I}\right)\left[T_\mathrm{C}^{*}\left(\varepsilon_\mathrm{I}\right)\left(1-t^{2}\right)\right]^{2}\left[B_\mathrm{C2}^{*}\left(T,\varepsilon_\mathrm{I}\right)\right]^{n-3}b^{p-1}\left(1-b\right)^{q} \qquad \text{(A20)}$$

$$B_\mathrm{C2}^{*}\left(T,\varepsilon_\mathrm{I}\right)=B_\mathrm{C2}^{*}\left(0,\varepsilon_\mathrm{I}\right)\left(1-t^{\nu}\right) \qquad \text{(A21)}$$

$$\left(\frac{A\left(\varepsilon_\mathrm{I}\right)}{A\left(0\right)}\right)^{1/u}=\left(\frac{B_\mathrm{C2}^{*}\left(0,\varepsilon_\mathrm{I}\right)}{B_\mathrm{C2}^{*}\left(0,0\right)}\right)^{1/w}=\frac{T_\mathrm{C}^{*}\left(\varepsilon_\mathrm{I}\right)}{T_\mathrm{C}^{*}\left(0\right)} \qquad \text{(A22)}$$

$$\frac{B_\mathrm{C2}^{*}\left(0,\varepsilon_\mathrm{I}\right)}{B_\mathrm{C2}^{*}\left(0,0\right)}=1+c_{2}\varepsilon_\mathrm{I}^{2}+c_{3}\varepsilon_\mathrm{I}^{3}+c_{4}\varepsilon_\mathrm{I}^{4}\,. \qquad \text{(A23)}$$

where: J_C: engineering critical current density ($\mathrm{Am^{-2}}$), defined as the critical current divided by the total cross-sectional-area of the wire.

$\varepsilon_\mathrm{I}=\varepsilon_\mathrm{A}-\varepsilon_\mathrm{M}$. ε_I: intrinsic strain. ε_A: applied strain. ε_M: applied strain at the peak (all in units of percent).

T_C^{*}: effective critical temperature (K).

$t=T/T_\mathrm{C}^{*}$: reduced temperature.

B_C2^{*}: effective upper critical field (T).

$b=B/B_\mathrm{C2}^{*}$: reduced magnetic field.

The simplified interpolative scaling law involves 13 parameters, the values of which are presented below for a number of Nb_3Sn strands and a Nb_3Al strand (for J_C defined at 10 $\mu\mathrm{Vm^{-1}}$).

For parameterising partial datasets, universal values are used for some of the parameters, as described in Ref. A1. For example, where only variable-strain limited-variabl;e-field data at 4.2 K are available, the values $p=0.5$, $q=2$, $n=2.5$, $\nu=1.5$, $w=2.2$, and $T_\mathrm{C}^{*}\left(0\right)$ = 17.5 K are used (see Table 3 below—Furukawa strand).

1 EM-LMI ITER Nb_3Sn (diameter: 0.81 mm)

Data range parameterised: $0.5\ \mathrm{T} \le B \le 23\ \mathrm{T}$, $4.2\ \mathrm{K} \le T \le 12\ \mathrm{K}$, $-0.81\% \le \varepsilon_A \le 0.61\%$.
RMS difference between measured and calculated values: 1.4 A.

p	Q	n	v	w	u	ε_M (%)
0.4741	1.953	2.338	1.446	1.936	–0.056	0.2786
$A(0)$ $(\mathrm{Am^{-2}T^{3-n}K^{-2}})$	$T_C^*(0)$ (K)	$B_{C2}^*(0,0)$ (T)	c_2	c_3	c_4	
2.446×10^7	16.89	28.54	−0.7697	−0.4913	−0.0538	

2 Vac ITER Nb_3Sn (diameter: 0.81 mm)

Data range parameterised: $0.5\ \mathrm{T} \le B \le 23\ \mathrm{T}$, $4.2\ \mathrm{K} \le T \le 12\ \mathrm{K}$, $-1.22\% \le \varepsilon_A \le 0.73\%$.
RMS difference between measured and calculated values: 2.5 A.

p	Q	n	v	w	u	ε_M (%)
0.4625	1.452	2.457	1.225	2.216	0.051	0.3404
$A(0)$ $(\mathrm{Am^{-2}T^{3-n}K^{-2}})$	$T_C^*(0)$ (K)	$B_{C2}^*(0,0)$ (T)	c_2	c_3	c_4	
9.460×10^6	17.58	29.59	−0.6602	−0.4656	−0.1075	

3 Furukawa ITER Nb_3Sn (diameter: 0.81 mm)

Data range parameterised: $0.5\ \mathrm{T} \le B \le 15\ \mathrm{T}$, $T = 4.2\ \mathrm{K}$, $-1.22\% \le \varepsilon_A \le 0.73\%$.
RMS difference between measured and calculated values: 2.7 A.

p	q	n	v	w	u	ε_M (%)
$0.5^\dagger$	$2^\dagger$	$2.5^\dagger$	$1.5^\dagger$	$2.2^\dagger$	$0^\dagger$	0.3152
$A(0)$ $(\mathrm{Am^{-2}T^{3-n}K^{-2}})$	$T_C^*(0)$ (K)	$B_{C2}^*(0,0)$ (T)	c_2	c_3	c_4	
1.112×10^7	$17.5^\dagger$	30.90	−0.6451	−0.4192	−0.0814	

†Universal values.

4 Sumitomo ITER Nb$_3$Al (diameter: 0.81 mm)

a) Data range parameterised: 0.5 T $\le B \le$ 15 T, 4.2 K $\le T \le$ 14 K, $-1.96\% \le \varepsilon_A \le 0.73\%$.
RMS difference between measured and calculated values: 1.1 A.

p	q	n	v	w	u	ε_M (%)
0.6973	2.671	2.651	1.269	1.933	−0.102	0.1609

$A(0)$ (Am^{-2}T^{3-n}K^{-2})	$T_C^*(0)$ (K)	$B_{C2}^*(0,0)$ (T)	c_2	c_3	c_4
2.564×10^7	15.68	26.81	−0.1050	−0.0074	0.0065

The following represent fits over reduced ranges, as performed in Ref A2:

b) Data range parameterised: 0.5 T $\le B \le$ 15 T, 4.2 K $\le T \le$ 10 K, $-0.73\% \le \varepsilon_A \le 0.73\%$.
RMS difference between measured and calculated values: 0.8 A.

p	q	n	v	w	u	ε_M (%)
0.7292	2.762	2.632	1.258	2.021	−0.687	0.1479

$A(0)$ (Am^{-2}T^{3-n}K^{-2})	$T_C^*(0)$ (K)	$B_{C2}^*(0,0)$ (T)	c_2	c_3	c_4
2.869×10^7	15.67	27.23	−0.1116	0.0116	0.0264

c) Data range parameterised: 0.5 T $\le B \le$ 15 T, 4.2 K $\le T \le$ 10 K, $-1.96\% \le \varepsilon_A < -0.73\%$.
RMS difference between measured and calculated values: 0.8 A.

p	q	n	v	w	u	ε_M (%)
0.4666	2.191	2.940	1.204	1.884	−0.107	0.1609

$A(0)$ (Am^{-2}T^{3-n}K^{-2})	$T_C^*(0)$ (K)	$B_{C2}^*(0,0)$ (T)	c_2	c_3	c_4
7.365×10^6	16.52	26.81	−0.2326	−0.1275	−0.0229

5 OST Nb_3Sn (diameter: 0.5 mm)

Data range parameterised: 0.5 T $\leq B \leq$ 15 T, 4.2 K $\leq T \leq$ 12 K, $-1.22\% \leq \varepsilon_A \leq 0.49\%$.
RMS difference between measured and calculated values: 1.7 A.

p	q	n	ν	w	u	ε_M (%)
0.4763	2.150	3.069	1.240	2.545	−0.912	0.2421

$A(0)$ (Am^{-2}T^{3-n}K^{-2})	$T_C^*(0)$ (K)	$B_{C2}^*(0,0)$ (T)	c_2	c_3	c_4
6.417×10^6	18.00	29.17	−0.6457	−0.4514	−0.1009

Ref. A1: D. M. J. Taylor and D. P. Hampshire, Phys. Rev. B (In progress September 2004)
Ref. A2: Keys, N. Koizumi, and D. P. Hampshire, Supercond. Sci. Tech. **15**, 991 (2002)