The Magnetic Field, Temperature and Strain Dependence of the Critical Current of a Nb_3Sn Strand Using a Six Free-Parameter Scaling Law

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Abstract—**We report comprehensive data for the magnetic field ($B < 14.5$ T), temperature (4.2 K $\leq T \leq$ 14 K) and uniaxial applied strain dependence ($-1\% < \varepsilon_A < 0.35\%$) of the critical current density (J_C) of an advanced internal-tin Nb_3Sn strand developed as part of the International Thermonuclear Experimental Reactor (ITER) program manufactured by Luvata PORI. A very short heat-treatment was used so that J_C of the PORI Nb_3Sn strand was ~40% lower than optimized values. Despite the low J_C, we find that the normalized strain sensitivity of the critical current at 14 T and 4.2 K is similar to other advanced internal-tin Nb_3Sn strands. By assuming that the strain dependence of the normalized effective upper critical field is similar to that specified for other advanced strands, the $J_C(B, T, \varepsilon)$ data of the PORI strand can be accurately parameterized using a scaling law with just 6 free parameters. Hence the results presented here on a low J_C strand provide evidence that the 6 free parameter scaling law can be expected to accurately characterize a wide range of advanced internal-tin Nb_3Sn strands. The index of transition or n value is described by a modified power law of the form $n = 1 + rI_C^s$, where r and s are approximately constant with values 3.15 and 0.36 respectively.**

Index Terms—**Critical current density, Nb_3Sn strand, scaling law, uniaxial strain.**

I. INTRODUCTION

THERE HAS BEEN the very fast development of Nb_3Sn strands in the last decade due to the requirement for large superconducting magnets—particularly motivated by the International Thermonuclear Experimental Reactor (ITER) project [1], [2]. The improvement has been focused on increasing the critical current density (J_C) and decreasing the cost [3]. With high critical current density (non-Cu J_C 800–1200 A $\cdot$ mm^{-2} at 4.2 K and 12 T) and acceptable ac losses (~ 600 kJ $\cdot$ m^{-3}), the use of advanced internal-tin Nb_3Sn strands is very attractive. As a result, significant effort has transferred from producing bronze-route strands (where non-Cu $J_C < 800$ A $\cdot$ mm^{-2}) to advanced internal-tin strands. The J_C (and I_C) versus magnetic field with zero applied strain of these two types of strands are shown in Fig. 1. The differences between the J_C (and I_C) values over a wide field range are clear.

It is well established that the effects of strain on technological Nb_3Sn strands must be characterized over a wide magnetic field and temperature range [4]–[11], because large stresses are unavoidable in large magnets originating from the differential thermal contraction between the components of the magnets during the process of cool-down and also the large Lorentz forces produced during high field operation. In this paper we present comprehensive $J_C(B, T, \varepsilon)$ data as a function of magnetic field ($B < 14.5$ T), temperature (4.2 K $\leq T \leq$ 14 K) and uniaxial applied strain ($-1\% < \varepsilon_A < 0.35\%$) for an advanced internal-tin PORI Nb_3Sn strand developed for the ITER project. By using a short heat-treatment, J_C is significantly reduced from optimum values which allows us to assess whether we can expect the 6 free parameter scaling law recently proposed [12] for advanced internal-tin strands to be of general validity for strands reacted with different heat-treatments.

II. EXPERIMENTAL PROCEDURE

A. Sample and Heat Treatment

The PORI strand, of diameter 0.81 mm, investigated in this paper was manufactured by Luvata PORI using sub-elements developed for the KSTAR project [13]. The strand was heat-treated in an argon atmosphere on oxidized stainless-steel mandrels in a three-zone furnace, with an additional thermocouple positioned next to the sample in order to monitor and control the temperature using the heat-treatment: 210°C for 48 h; 400°C for 48 h; 640°C for 60 h (all the ramp rates were 10°C $\cdot$ h^{-1}). This heat-treatment time is much shorter and hence the I_C much lower than that for maximum/optimum values [12]. Nevertheless, the I_C for the PORI strand is still significantly higher than that of bronze-route (Vac and Furukawa) strands as seen in Fig. 1. After reaction, the wires were etched in hydrochloric acid to remove the chrome and transferred to nickel-plated Ti-6Al-4V helical springs, to which they were attached using copper plating and soldering. An optimized helical spring [14], [15] was used for the $J_C(B, T, \varepsilon)$ measurements.

B. Apparatus and Techniques

After the PORI strand was attached to the spring, it was mounted onto our purpose-built $J_C(B, T, \varepsilon)$ probe [7], [10], [16]. The critical current measurements were carried out in Durham in magnetic fields up to 14.5 Tesla. Throughout this paper, the engineering J_C is quoted, which is defined as I_C divided by the entire cross-sectional area of the strand. For variable strain measurements, the spring is twisted to apply the strain to the wire via a worm-and-wheel system. For variable-temperature measurements above 4.2 K, the probe
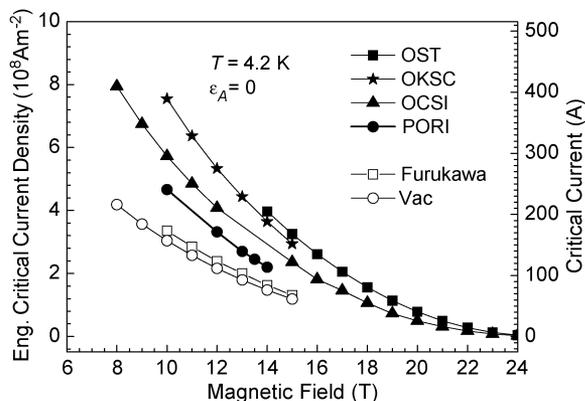
Fig. 1. Engineering critical current density J_C (and critical current I_C) versus magnetic field for Nb_3Sn strands made by different manufacturers. The Vac (now known as EAS) and Furukawa strands are bronze-route. OST, OKSC, OCSI and PORI strands are advanced internal-tin.

Fig. 2. Engineering critical current density (and critical current) of PORI strand as a function of applied strain at 4.2 K in magnetic fields from 10 and 14 T with increments (ΔB) 0.5 T. The solid lines are provided by the Durham scaling law (with 6 free parameters).

provides a vacuum chamber around the sample and the temperature is maintained during the measurement using three sets of independently controlled Cernox thermometers and constantan wire heaters distributed to produce a uniform temperature profile along the turns of the spring. For measurements at 4.2 K, the sample is in direct contact with the liquid helium.

At specified values of magnetic field, temperature and strain, measurements are made of the voltage (V) across sections of the strand as a function of the current through it. The voltage noise is typically a few nV predominantly due to the Johnson noise from the voltage leads. The voltage across a section of the wire was measured using a nanovolt amplifier and a digital voltmeter and the current was measured using a four-terminal standard resistor. The electric field criterion used for J_C is $10\ \mu V \cdot m^{-1}$ and the index of transition or n-value is calculated using the power-law expression $E \propto J^n$ with E in the range between 10 and $100\ \mu V \cdot m^{-1}$. Details of the experimental apparatus and techniques have been provided previously [15], [17], [18].

III. RESULTS

Fig. 2 shows J_C (and I_C) as a function of applied strain at 4.2 K in magnetic fields from 10 T to 14 T with the increment in field of 0.5 T. The well-known inverted quasi-parabolic behavior for J_C as a function of strain is very clear and the critical role of strain can be seen from noting that $\pm 0.5\%$ strain (around 4.2 K and 12 T) can decrease J_C by $\sim 50\%$. Fig. 3 provides a comparison between the normalized strain dependence of the critical current for the PORI strand investigated in this work and other optimized advanced internal-tin strands (OST, OKSC and OCSI) reported in the literature [12]. It can be seen that although the magnitude of I_C is significantly different for each of these four strands, to within the accuracy of the measurements, the strain dependence is very similar.

The Nb_3Sn Cable-In-Conduit-Conductors (CICC) used for the ITER magnet systems operate in supercritical helium at temperatures above 4.2 K. Fig. 4 shows J_C (and I_C) as a function
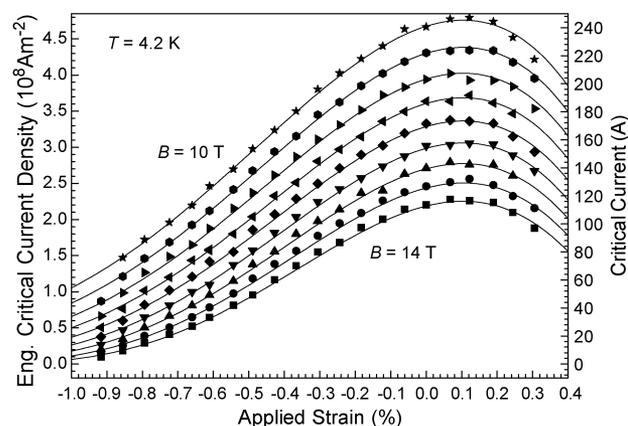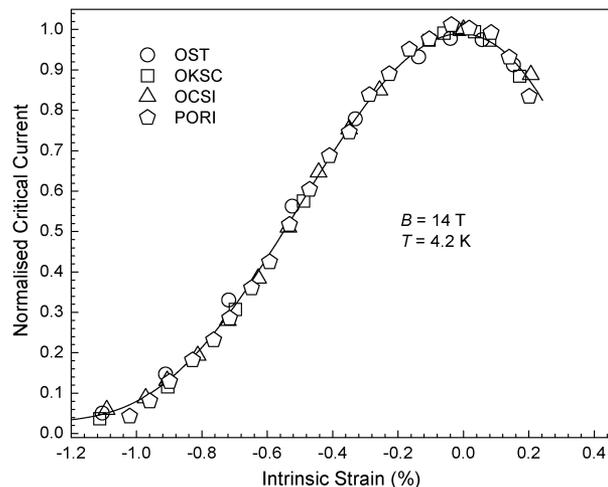
Fig. 3. The normalized critical current at 4.2 K as a function of intrinsic strain for OST ($I_{Max} = 219.6$ A), OKSC ($I_{Max} = 193.4$ A), OCSI ($I_{Max} = 149.6$ A) (taken from [12]), and the PORI ($I_{Max} = 116.1$ A) strand reported in this work. The solid line is a guide to the eye.

of applied strain from 6 K to 12 K in different magnetic fields. Limited 14 K data were also measured (not displayed here). The strong magnetic field, temperature and strain dependence of J_C (and I_C) is clearly observed in Figs. 1–4. In Figs. 2 and 4, the data points are the experimental measurements, and the solid lines are provided by the Durham scaling law with 6 free parameters which are provided in Table I. The details of the scaling law will be discussed in the Section IV.

The $E-J$ characteristics of the superconducting strands were characterized using the power law expression [19]–[21] $E \propto J^n$. The n-value characterizes the sharpness of the $E-J$ transition in technological superconductors [21]–[24]. The n-values for the PORI strand were obtained from $E-J$ characteristics over the technological range of interest from 10 to $100\ \mu V \cdot m^{-1}$ and are presented as a function of critical current at different magnetic fields, temperatures and strains in Fig. 5.
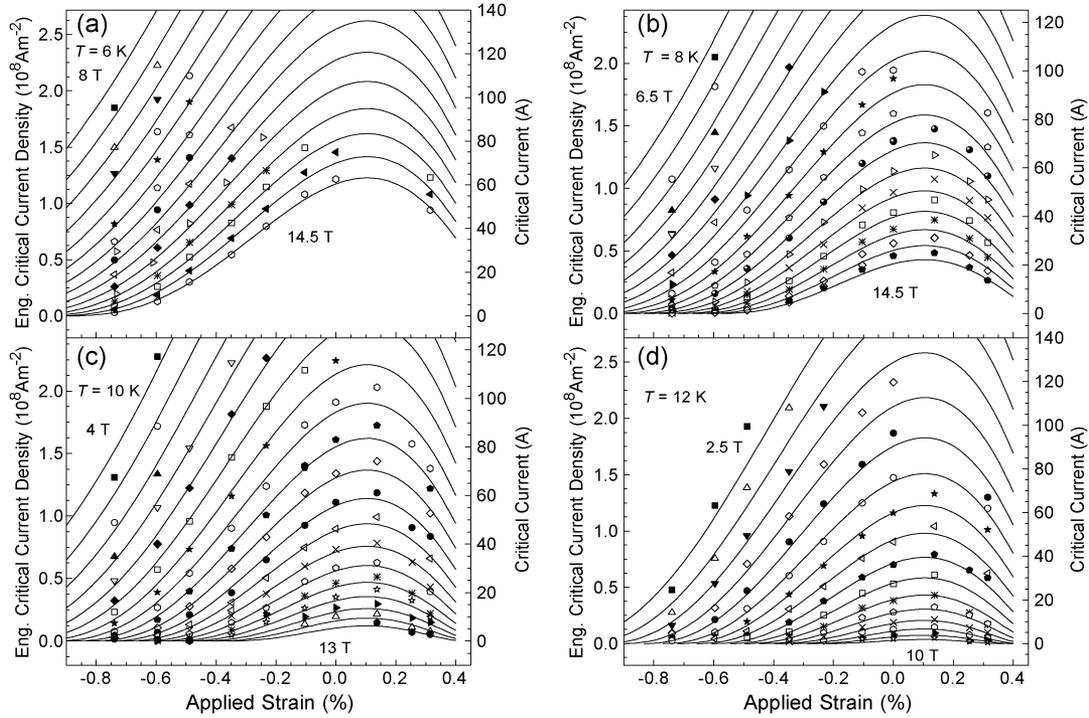
Fig. 4. Engineering critical current density (and critical current) as a function of applied strain at magnetic fields shown. Panels (a)–(d) are at temperatures of 6 K, 8 K, 10 K and 12 K respectively. The lines are provided by the Durham scaling law using (6 free) parameters from Table I.

TABLE I
PARAMETERS OF DURHAM SCALING LAW FOR PORI STRAND: SIX
PARAMETERS ARE FREE AND SEVEN, GIVEN IN BOLD, WERE NOT VARIED
IN THE FITTING PROCEDURE

Universal Values	c_2	c_3	c_4	n	ν	w	u
	-0.77462	*-0.59345*	*-0.13925*	*2.5*	*1.5*	*2.2*	*0*

$A(0)$ (Am^{-2}T^{3-n}K^{-2})	$T_C^*(0)$ (K)	$B_{C2}^*(0,0)$ (T)	p	q	ε_M (%)
3.644×10^7	16.63	30.01	1.0129	2.653	0.1044

Fig. 5. $n-1$ as a function of critical current for PORI strand.

IV. ANALYSIS

A. Variable-Strain Critical Current Data Analysed Using Scaling Laws

Extensive work on a range of different Nb₃Sn strands fabricated using different manufacturing techniques has lead to a scaling law for $J_C(B,T,\varepsilon)$ of the form [7]

$$J_C(B,T,\varepsilon_I) = A(\varepsilon_I)\left[T_C^*(\varepsilon_I)(1-t^2)\right]^2$$
$$\times \left[B_{C2}^*(T,\varepsilon_I)\right]^{n-3} b^{p-1}(1-b)^q \quad (1)$$

$$B_{C2}^*(T,\varepsilon_I) = B_{C2}^*(0,\varepsilon_I)(1-t^\nu) \quad (2)$$

$$\left(\frac{A(\varepsilon_I)}{A(0)}\right)^{1/u} = \left(\frac{B_{C2}^*(0,\varepsilon_I)}{B_{C2}^*(0,0)}\right)^{1/w} = \frac{T_C^*(\varepsilon_I)}{T_C^*(0)} \quad (3)$$

where J_C is the engineering critical current density, ε_A is the applied strain, $\varepsilon_I = \varepsilon_A - \varepsilon_M$ is the intrinsic strain, ε_M is the applied strain at the peak in J_C [25]–[27], T_C^* is the effective

critical temperature, $t = T/T_C^*$ is the reduced temperature, B_{C2}^* is the effective upper critical field and $b = B/B_{C2}^*$ is the reduced field. The number of free parameters can be reduced by using universal values for n, ν, w and u [7], [8].

Recently we reported $J_C(B,T,\varepsilon)$ measurements made on 3 optimized advanced internal-tin Nb₃Sn strands in magnetic fields up to 28 Tesla in Grenoble where the upper critical fields were measured directly [12]. Based on those measurements, we proposed a universal strain dependence for the normalized effective upper critical field of advanced strands at $T = 0$ of the form [12]

$$\frac{B_{C2}^*(0,\varepsilon_I)}{B_{C2}^*(0,0)} = 1 - 0.77462\cdot\varepsilon_I^2 - 0.59345\cdot\varepsilon_I^3 - 0.13925\cdot\varepsilon_I^4 \quad (4)$$

Equations (1)–(4) lead to a $J_C(B,T,\varepsilon)$ scaling law for advanced internal-tin strands with just six free parameters. The six free parameters for the PORI strand are presented in Table I together

with the other seven parameters that were not varied in the fitting process.

There is good agreement between the experimental $J_C(B, T, \varepsilon)$ data and the solid lines for the PORI strand shown in Figs. 2 and 4. The data at the lowest reduced fields shows the largest deviation from the scaling law. This is because the reduced field dependence in (1), which includes the parameters p and q, is a high-field expression (first proposed in the Fietz-Webb scaling law [28]). It is not sufficiently general to describe the field dependence of over the entire field range. In low fields non-linear terms (in the free-energy [29] and hence required in a pinning description of $J_C(B, T, \varepsilon)$ [30]) must be included for a more accurate parameterization. Nevertheless, the root-mean-square (RMS) difference between the measured and calculated values of I_C is about ~ 3 A over the whole measured range. This agreement can be further improved if necessary by using 9 free parameters (not provided here), which gives an RMS difference of ~ 2.4 A.

B. n-Values

The index of transition or n-value data are characterized using an empirical modified power law

$$n(B, T, \varepsilon_I) = 1 + r(T, \varepsilon_I) [I_C(B, T, \varepsilon_I)]^{s(T, \varepsilon_I)} \qquad (5)$$

As observed previously [20], $s(T, \varepsilon_I)$ is approximately a constant for all temperatures and applied strains and $r(T, \varepsilon_I)$ only weakly depends on strain [12], [31]. From Fig. 5 we find that r and s are 3.15 and 0.36 respectively, which are values similar to those of other advanced internal-tin strands [12]. We attribute the deviation from linearity at the highest currents in Fig. 5 to heating during the transition.

V. DISCUSSION

The early development of high J_C strands for magnet applications involved doping binary Nb_3Sn with elements such as Ta and Ti to increase the upper critical field (B_{C2}). Improvements of ~ 7 T were achieved at the expense of a stronger strain dependence for B_{C2} which can be explained using standard microscopic theory [7]. It follows that strands made using different fabrication routes, which lead to Nb_3Sn with different Sn content will have different reversible strain dependencies—consistent with the differences observed recently between bronze route strands and advanced internal-tin strands [7]. It is well-known that during the heat treatment of strands, the average Sn content and the morphology of the first Nb_3Sn that is formed is quite different to the average properties of the final Nb_3Sn layer [32], [33]. This opens the possibility that even strands fabricated in a similar way may not have a similar strain dependence if the heat-treatment is significantly different. However, in this work we have found that an advanced strand reacted for a relatively short time has a similar normalized strain dependence for J_C and $B_{C2}^*(0, 0)$ to advanced strands optimized for high J_C. Although J_C is much lower for the PORI strand than the optimized advanced strands in the literature, the $B_{C2}^*(0, 0)$ values are similar [12]. Reliable comparisons of the (fitting) parameter $T_C^*(0)$ for different advanced strands are more problem-

atic. Comparing fits to data on different strands, the type of fitting and quantity of data used at high temperature can affect $T_C^*(0)$. Equally since the distribution of the critical temperature present (across the compositional variation) in a filament can be large, a measurement of the critical temperature can be strongly dependent on how the measurement is made (e.g. resistive/percolative or magnetic/screening). Fortunately there is detailed thesis work by Naus [34] that suggests the critical temperature only changes by about 0.3 K between 60 h and 300 h in advanced internal-tin strands reacted at 650°C. Hence we conclude that the long heat-treatment's primary effect, from the perspective of the reversible strain properties of J_C, is to increase the cross-sectional area of superconductor carrying the current without changing the intrinsic critical parameters very significantly and suggest that the normalized strain dependence observed in advanced internal-tin strands can be considered intrinsic to the composition of the Nb_3Sn layer that is produced.

VI. CONCLUSION

In this paper we have reported comprehensive variable magnetic field, variable temperature and variable strain J_C data for an advanced internal-tin Nb_3Sn ITER strand (PORI) manufactured by Luvata. The relatively short heat treatment used for the PORI strand leads to significantly lower critical current density than optimized values. However the normalized strain dependence of the critical current at 14 Tesla and 4.2 K is similar to other strands made by an internal-tin process [12]. The parameterization of the data shows that a scaling law can accurately parameterize the $J_C(B, T, \varepsilon)$ data using just six free parameters. This scaling law is based on assuming a universal behavior of the normalized effective upper critical field for the advanced internal-tin Nb_3Sn strands and provides a framework for comparing partial data sets from different laboratories.

This work shows that the reversible strain dependence of the normalized critical current at 4.2 K and 14 T and the upper critical field [12] are properties intrinsic to advanced internal-tin strands—characteristic of the fabrication route and (beyond 60 h) insensitive to the duration of the heat treatment.

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