Geometrical surface pinning in the nonlinear AC susceptibility of HTS tapes

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Abstract. The analytic expression for the nonlinear magnetic susceptibility of a thin disk derived from Bean's model with a uniform $J_{\rm C}$ for $B \parallel \hat{n}$ predicts a peak-peak ratio of 7.1 (5.4) for the real (imaginary) components of the third harmonic χ_3 susceptibility. Our measurements show a peak-peak ratio closer to 1.1 in the real component and a noise limited lower estimate of \approx 7 in the imaginary component. The anomalous third harmonic can be explained by the presence of a geometrical surface barrier for flux entry and exit which we have included as a small region of enhanced current density, $J_{\rm C,G}$, close to the surface of the disk. The geometrical surface current density is predicted to be approximately equal to $J_{\rm C,G} \approx 2B_{\rm C1}/\mu_0 d$. In 2G HTS tapes with $d \approx 1\mu$ m and $B_{\rm C1} \approx 5$ mT this is $J_{\rm C,G} \approx 8$ GAm⁻². We have measured both $J_{\rm C,B}$ and $J_{\rm C,G}$ using the nonlinear susceptibility of a Superpower tape without artificial pinning in fields orthogonal to the tape up to 35T. We find a geometrical surface current of $J_{\rm C,G} \approx 10$ GAm⁻² and an average critical current density $J_{\rm C}$ which is in good agreement with transport measurements performed on material from the same reel of tape.

1. Introduction

The critical current density, $J_{\rm C}$ of state-of-the-art High Temperature Superconducting (HTS) tapes is currently one or two orders of magnitude below the theoretical maximum depairing current predicted by Ginzburg-Landau theory in high magnetic fields [1] [2]. The drive for a higher $J_{\rm C}$ plays a crucial role in the development of high field magnets for both research and commercial purposes. Transport measurements of $J_{\rm C}$ can be time consuming and often require a reasonably large bore size (≈ 40 mm), both of which are serious limitations if one wishes to perform $J_{\rm C}$ measurements at the highest fields technologically available. Magnetic measurements present a viable alternative method for measuring $J_{\rm C}$ quickly in very high fields but they require an accurate functional form relating the measured magnetisation M(B) to the critical current density.

Typically [3–6], the functional form used for M(B) in magnetic measurements of $J_{\rm C}$ is that of Bean's model [7] with a uniform $J_{\rm C}$, in either the thin strip [8] or thin disk geometry [9]. In this paper we study the magnetisation of square sections of tape, simulations of which [10] suggest the magnetisation differs from the thin disk by less than 0.2% over the entire magnetisation curve.

The thickness averaged current density of a zero field cooled (ZFC) thin disk of thickness d and radius R with a uniform $J_{\rm C}$ in an applied field B was first presented by Mikheenko &

Kuzovlev [9]

$$\frac{1}{d} \int_{\frac{d}{2}}^{\frac{d}{2}} J(B,\rho,z) dz \equiv \bar{J}(B,\rho) = \begin{cases} J_{\rm C} \frac{2}{\pi} \tan^{-1} \left(\frac{\tanh(\frac{2B}{\mu_0 J_{\rm C} d})\rho}{\sqrt{a^2 - \rho^2}} \right) & \rho \le a \\ J_{\rm C} & \rho > a \end{cases}$$
(1)

Where $a(B)/R = 1/\cosh(\frac{2B}{\mu_0 J_C d})$. Simulations by Brandt [11] considered the variation in current density across the thickness of the disk and hysteretic flux motion such that the current density is only either J = 0 or $J = J_C$ (as per Bean's model). He found solutions consistent with equation (1) and Maxwell's equations, provided that the region of zero current density and J_C are delineated by the surface

$$z(\rho, B) \approx \frac{d}{\pi} \arccos \frac{\rho}{a(B)}$$
 (2)

Hence using equation (1), the magnetisation of a thin disk is given by

$$\mathbf{M}_{\text{bulk}}(\mathbf{B}) \equiv \frac{1}{2V} \int_{V} \mathbf{r} \times \mathbf{J} d^{3} \mathbf{r} = -\frac{2J_{\text{C}} R \hat{\mathbf{z}}}{3\pi} \left[\cos^{-1} \left(\frac{a}{R} \right) + \left(\frac{a}{R} \right)^{2} \sinh \left| \frac{2B}{\mu_{0} J_{\text{C}} d} \right| \right]$$
(3)

From which the magnetisation during a complete AC field cycle can be determined through the general formulation [11–14]

$$\mathbf{M}_{\downarrow}(\mathbf{B}(\omega t)) = \mathbf{M}(\mathbf{B}_{max}) - 2\mathbf{M}\left(\frac{\mathbf{B}_{max} - \mathbf{B}(\omega t)}{2}\right)$$
$$\mathbf{M}_{\uparrow}(\mathbf{B}(\omega t)) = -\mathbf{M}(\mathbf{B}_{max}) + 2\mathbf{M}\left(\frac{\mathbf{B}_{max} + \mathbf{B}(\omega t)}{2}\right)$$
(4)

for the increasing (\uparrow) and decreasing (\downarrow) applied field.

For an AC field $\mathbf{B}(\omega t) = \mathbf{B}_1 \cos(\omega t)$ the harmonics of the nonlinear susceptibility tensor are defined as

$$\chi_{n,ij}' \equiv \frac{\mu_0}{\pi B_{1,j}} \int_0^{2\pi} M_i(\omega t) \cos(n\omega t) d(\omega t)$$

$$\chi_{n,ij}'' \equiv \frac{\mu_0}{\pi B_{1,j}} \int_0^{2\pi} M_i(\omega t) \sin(n\omega t) d(\omega t)$$
(5)

In general we cannot rule out the possibility of magnetisation currents flowing in planes perpendicular to the applied field, and for tapes in tilted magnetic fields where there is a complex intertwined network of pancake [15] and Josephson vortices [16], the susceptibility tensor may have off-diagonal components. In this work, we have applied an AC field parallel to the tape normal and the applied DC field, and found that the induced net moment is parallel to within 4°. For the purposes of this paper, we will assume the magnetic moment is parallel to both the applied DC and AC fields (i.e. $\mathbf{M} \parallel \mathbf{B} \parallel \mathbf{z}$). Here we also extend analytic consideration of the current density equation (1) and magnetisation equation (3) to include a region close to the surface of the disk with an enhanced current density, $J_{C,G}$, which is greater than the bulk current density, $J_{C,B}$. Following Zeldov [17], the physical interpretation of this is the existence of a geometric barrier for flux entry and exit due to a finite B_{C1} in which the magnetisation currents flow at $J_{C,G} \approx 2B_{C1}/\mu_0 d$. For a superconductor of thickness $d = 1\mu m$ with a lower critical field $B_{C1} \approx 5 \text{mT}$ [18] the predicted geometrical surface current density is $J_{C,G} \approx 8 \text{GAm}^{-2}$ flowing over a range $\sim d/2$ from the surface.

The paper is structured as follows: In Section 2 we provide typical data and see by inspection that $J_{C,G}$ cannot be spatially uniform in the film. Then we identify the qualitative features

observed in experimental data which are better explained through the inclusion of a geometrical surface barrier (figures 1 and 2). In Section 3 we detail the mathematical extension to equation (1) and in Section 4 we present the experimentally determined $J_{\rm C}(B,T)$ behaviour for the Superpower non-AP tape in fields up to 35T. We compare these magnetic results with transport measurements performed on the same reel of tape and obtain a Kramer fit [19] for the pinning force density. In Section 5 we make some concluding remarks regarding the implications of geometrical surface currents for magnetic measurements of $J_{\rm C}$ in 2G HTS tapes.

2. Experimental Methodology and Harmonic data

We measured the AC susceptibility of a Superpower HTS Tape without artificial pinning in fixed DC fields up to 35T at LNCMI, Grenoble while changing the temperature. The AC excitation field had a frequency of 777Hz and an amplitude of 0.5mT. Equation (3) predicts that for a spatially uniform $J_{\rm C}$, a peak-peak ratio of 7.1 (5.4) for the real (imaginary) components of the third harmonic χ_3 susceptibility should be observed. However, as shown in figure 1, we have observed an anomalous peak-peak ratio of 1.1 in the χ'_3 component and no observable 2nd peak in the χ''_3 component, from which we can estimate a noise limited lower bound of ≈ 7 on the peak-peak ratio. This anomalous peak behaviour in χ_3 cannot be explained using equation (3). As shown below, we find that the addition of a geometrical barrier provides a way to describe the experimental harmonic data and leads to an average critical current $J_{\rm C}(B,T)$ in good agreement with transport data.



Figure 1. The experimentally observed (a) real and (b) imaginary and the analytic (c) real and (d) imaginary components of the third harmonic of the nonlinear susceptibility. The analytic susceptibilities are shown for a range of geometrical surface current densities, $J_{C,G}$.



Figure 2. The analytic fundamental susceptibility, both real (χ'_1) and imaginary (χ''_1) components, for a range of geometrical surface currents.

3. AC Susceptibility of Thin Disks with an Enhanced Surface Current

In type II superconductors where $B_{C1} \neq 0$, there exists a barrier for flux entry and exit which the flux lines must overcome. For flux lines parallel to a superconducting-normal boundary this is described well by the Bean-Livingston Barrier [20]. For flux lines attempting to penetrate through a sharp corner, there is a geometric barrier which prevents entry into the bulk until the elastic force on the deformed flux line overcomes the Lorentz force preventing entry [17]. Here we present an extension to equation (1) for the current density and magnetisation of a thin superconducting disk with a geometric barrier.

3.1. Inclusion of an Enhanced Geometrical Surface Current:

We can introduce a surface region with an enhanced current density, $J_{C,G}$. The thickness averaged current density for a thin disk with an enhanced surface current can be split into two regimes, $B \leq B_S$, where the applied field is too small to penetrate through the geometrical surface barrier. $B > B_S$, where the applied field is high enough to penetrate through through the barrier into the bulk of the superconductor. B_S is the surface penetration field. The extent of the geometrical barrier is given a characteristic length Λ transcendentally through

$$1 - \frac{\Lambda}{R} = \frac{1}{\cosh\left(\frac{B_{\rm S}}{\mu_0 J_{\rm C,G}\Lambda}\right)}\tag{6}$$

where given the line-energy considerations for a vortex deformed around the disk, it is appropriate to choose a value of $\Lambda = d/2$ [17]. In this paper we require that the current densities induced as a result of the surface pinning are described by Bean's model, and are consistent with Maxwell's equations. Hence the surface pinning is characterised here by a geometrical current density flowing over a distance $\Lambda = d/2$. For $d \ll R$, $B_{\rm S} \approx \frac{\mu_0 J_{\rm C,G} d}{2} \sqrt{\frac{d}{R}}$.

3.1.1. $B \leq B_{\rm S}$:

For increasing field, when $B \leq B_S$, flux lines are prevented from entering the bulk of the disk by the geometrical surface barrier and the thickness averaged current density in the disk is

$$\bar{J}(\rho) = \begin{cases} J_{C,G} & \rho > a \\ J_{C,G} \frac{2}{\pi} \tan^{-1} \left(\frac{\tanh(\frac{2B_0}{\mu_0 J_{C,G} d})\rho}{\sqrt{a^2 - \rho^2}} \right) & \rho \le a \end{cases}$$
(7)

Where the radius to which the vortices penetrate, a, is

$$\frac{a(B)}{R} \approx 1 - \frac{1}{2} \left(\frac{2B}{\mu_0 J_{\rm C,G} d}\right)^2 \tag{8}$$

and the magnetisation is

$$M(B) \approx -\frac{8RB}{3\pi\mu_0 d} \tag{9}$$

This linear magnetic response corresponds to a geometry dependent perfect diamagnetic susceptibility

$$\chi_0 \equiv \lim_{B \to 0} \left[\frac{\partial(\mu_0 M)}{\partial B} \right] = -\frac{8R}{3\pi d} \tag{10}$$

3.1.2. $B > B_S$:

If we increase the field further so that $B > B_S$, the applied field penetrates into the bulk of the disk. The thickness averaged current density and component of the magnetic field perpendicular to the plane of the disk are shown in figure 3 for a range of geometrical surface current densities $J_{C,G}$. The thickness averaged current density, $\bar{J}(\rho)$, is

$$\bar{J}(\rho) = \begin{cases} J_{\mathrm{C,G}} & \rho > R - d/2 \\ J_{\mathrm{C,B}} + \Delta J_{\mathrm{G}} \frac{2}{\pi} \tan^{-1} \left(\frac{\tanh(\frac{2B_{\mathrm{S}}}{\mu_0 J_{\mathrm{C,G}} d})\rho}{\sqrt{(R - d/2)^2 - \rho^2}} \right) & a \le \rho < R - d/2 \\ J_{\mathrm{C,B}} \frac{2}{\pi} \tan^{-1} \left(\frac{\tanh(\frac{2(B - B_{\mathrm{S}}(\Delta J_{\mathrm{G}}/J_{\mathrm{C,G}}))}{\mu_0 J_{\mathrm{C,B}} d})\rho}{\sqrt{a^2 - \rho^2}} \right) + \Delta J_{\mathrm{G}} \frac{2}{\pi} \tan^{-1} \left(\frac{\tanh(\frac{2B_{\mathrm{S}}}{\mu_0 J_{\mathrm{C,G}} d})\rho}{\sqrt{(R - d/2)^2 - \rho^2}} \right) & \rho < a \end{cases}$$

$$(11)$$

Where we have introduced ΔJ_G and by definition $J_{C,G} = J_{C,B} + \Delta J_G$. The radius to which the increasing field penetrates is



Figure 3. (a) The thickness averaged current density and (b) axial magnetic field strength for a thin disk with a geometrical surface current density $J_{C,G} = 100 \times J_{C,B}$.

And the magnetisation is found using the functional form for $M_{\text{bulk}}(B)$ in equation (3)

$$M(B) = M_{\text{bulk}} \left(B - B_{\text{S}} \left(\frac{\Delta J_{\text{G}}}{J_{\text{C,G}}} \right) \right) - \frac{8RB_{\text{S}}}{3\pi\mu_0 d}$$
(13)

The saturation magnetisation and field, respectively, are given by

$$M_{\rm sat} = -\frac{J_{\rm C,B}R}{3} \left(1 + \frac{8B_{\rm S}}{\pi\mu_0 J_{\rm C,B}d} \left(\frac{\Delta J_{\rm G}}{J_{\rm C,G}} \right) \right)$$

$$B_{\rm sat} = \frac{\mu_0 J_{\rm C,B}d}{2} \left(2 + \ln\left(\frac{2R}{d}\right) \right) + B_{\rm S} \left(\frac{\Delta J_{\rm G}}{J_{\rm C,G}}\right)$$
(14)

4. Analysis of Experimental Results

The magnetisation of a superconductor in a complete AC field cycle is obtained via equations (4) and (5). Figures 1 and 2 show the effect of an enhanced surface current on the 3rd harmonic, χ_3 , and fundamental, χ_1 , susceptibilities respectively. There are no large differences between the between the observed χ_1 susceptibility for both the real, χ'_1 , and imaginary, χ''_1 , components although there is a distinct shift of the entire diamagnetic transition towards a higher AC field strength. In contrast however, the third harmonic susceptibility, χ_3 , shows a strong dependence on $J_{C,G}$. Namely, the ratio of peak heights in the real (imaginary) component of χ_3 shifts from $7.1 \rightarrow 1$ (5.4 \rightarrow 8.1). Our observed data exhibit field independent peak-peak ratios of 1.1 (> 7) in the χ'_3 (χ''_3) component. These observed ratios cannot be explained with a uniform J_C throughout the disk and therefore we conclude that there is good evidence, from AC susceptibility data for geometrical surface pinning. Figure 4 show the surface and bulk critical current density calculated using equation (13).

Figure 4 shows the surface and bulk critical current densities calculated calculated from the harmonic susceptibility data without any a priori assumptions about the field and temperature dependence of either current density. These data show that most of the current in the tape is carried by the bulk material. We also assuming identify a field independent geometrical surface current density $J_{C,G} \approx 10 \text{GAm}^{-2}$. Figure 5(a) shows the average critical current J_C which we have calculated as a sum of the geometrical surface current density and the bulk current density weighted by the relative area of the surface region to the bulk. The average current density



Figure 4. (a) The measured bulk current density and (b) the measured geometrical surface current density at a range of temperatures from $0 \rightarrow 90$ K and DC fields from $0 \rightarrow 35$ T.



Figure 5. (a) The measured average current density from transport (\oplus) and susceptibility (\bullet) measurements and (b) a Kramer fit of the data indicating 2D bulk pinning as the dominant pinning mechanism.

is compared to transport measurements performed on the same reel of tape. The $J_{\rm C}$ data are also plotted on a normalised scaling law plot of the volume pinning force, $F_p \equiv J_{\rm C}B$, shown in figure 5(b) and fitted using the expression [19, 21]

$$F_p = Cb^p (1-b)^q \tag{15}$$

Where C is a temperature dependent prefactor, $b = B/B_{C2}$ is the reduced field and p and q are global constants. We have found $p = 0.51 \pm 7\%$ and $q = 2.3 \pm 4\%$ which suggests the dominant pinning mechanism in this tape is bulk pinning (p = 0.5, q = 2) [21]. Most of the pinning is most likely from a combination of internal twinning boundaries and defects as well as low angle grain boundaries.

5. Conclusions

Our measurements of the nonlinear AC susceptibility on a Superpower Non-AP tape show a peak-peak ratio in χ_3 which cannot be explained without the addition of a geometrical surface barrier for flux entry and exit. We have calculated a geometrical surface current $J_{C,G} \approx 10 \text{GAm}^{-2}$ which is in good agreement with the prediction that $J_{C,G} \approx 2B_{C1}/\mu_0 d \approx 8\text{GAm}^{-2}$ [17]. We have also obtained an estimate for the average critical current density including geometrical surface effects which is in good agreement with transport data performed on the same reel of tape and indicates that the dominant pinning mechanism for this tape is bulk pinning[21]. This is the first time such a barrier has been observed experimentally in 2G HTS tapes and its presence has several notable implications. Regarding transport measurements, while the geometrical surface barrier occupies only a small fraction (< 1%) of the cross sectional area of a tape, it may carry up to 5% of the total current and as a result demonstrates that if multifilamentary HTS tapes can be fabricated, one can expect them to carry significantly higher J_{C} values in high fields than is currently state of the art in tapes.

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