Unifying the strain and temperature scaling laws for the pinning force density in superconducting niobium-tin multifilamentary wires

Najib Cheggour[a] and Damian P. Hampshire
Superconductivity Group, Department of Physics, University of Durham, South Road, Durham DH1 3LE, England

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Systematic variable temperature measurements of the transport critical current density (J_c) tolerance to strain (ϵ), performed on a bronze processed niobium-tin multifilamentary wire in high magnetic fields up to 15 T, are reported. The results show that $B^*_{c2}(T,\epsilon)$, the field at which the pinning force density (F_p) extrapolates to zero, can be written as $B^*_{c2}(0,\epsilon)g[T/T^*_c(\epsilon)]$, where g is a function of the reduced temperature $T/T^*_c(\epsilon)$ and $T^*_c(\epsilon)$ is the temperature at which B^*_{c2} extrapolates to zero. We propose a magnetic field, temperature, and strain scaling law for F_p which unifies Ekin's strain scaling law and the Fietz–Webb variable temperature scaling law. It is of the form $F_p = J_c \times B = A(\epsilon)[B^*_{c2}(T,\epsilon)]^n b^p (1\text{-}b)^q$, where n, p, and q are constants, $A(\epsilon)$ is a function of strain alone, and b is the reduced field B/B^*_{c2}. [S0021-8979(99)04412-6]

I. INTRODUCTION

The enormous research activity into the properties of superconducting materials has lead to the rapid development of cryocooled magnet systems operating at temperatures above 4.2 K using low temperature[1] and high temperature superconductors.[2] In such systems, it is important to know the effect of strain, magnetic field, and temperature on the critical current density in the component wires or tapes. Furthermore, a detailed functional form for the critical current density (J_c) may help establish the mechanisms that determine the pinning of magnetic flux lines, and hence, aid further optimization of superconductors for high field applications. Fietz and Webb first measured a scaling law for the volume pinning force $F_p(=J_c \times B)$ in a series of niobium alloys.[3] Later, several authors[4–6] found that variable temperature J_c data can be described by a general form for F_p of

$$F_p(B,T) = A[B^*_{c2}(T)]^n b^p (1-b)^q, \tag{1}$$

where A, n, p, and q are constants, B^*_{c2} is the field at which F_p extrapolates to zero, and b is the reduced field B/B^*_{c2}. Since the critical current density in superconducting wires was found to depend on strain,[7] extensive experimental and theoretical work has been carried out to incorporate strain into the scaling law.[8–13] Ekin[9] reported a strain scaling law obtained at 4.2 K of the form

$$F_p(B,\epsilon) \propto [B^*_{c2}(4.2\ \mathrm{K},\epsilon)]^m b^p (1-b)^q. \tag{2}$$

The power m was found to be about 1 for niobium-tin (Nb_3Sn) although the power n in the Fietz–Webb law is typically around 2.5. In general, the values of n and m are not the same for a given material.[12] To date, no detailed variable temperature, variable strain J_c measurements have been made on high current density, technological superconducting wires. Theoretical work directed at unifying the Fietz–Webb variable temperature scaling law with Ekin's strain scaling law has yet to be supported by detailed experimental J_c data. Limited variable temperature, variable strain measurements have been reported using (pumped) cryogens[14,15] or samples with low critical currents.[16] However in this article, systematic transport $J_c(B,T,\epsilon)$ measurements on a bronze processed Nb_3Sn multifilamentary wire are reported throughout a wide temperature range. We address the functional form of $B^*_{c2}(T,\epsilon)$ and propose a unified scaling law that describes the temperature, strain, and field dependence of F_p. This scaling law eliminates the apparent inconsistency between the dependence of F_p on B^*_{c2} (i.e., the different values of n and m) found in the Fietz–Webb law and Ekin's law.

II. DESCRIPTION OF THE STRAIN PROBE

We have developed a probe to perform transport $J_c(B,T,\epsilon)$ measurements.[17] The strain is applied to the sample using the technique developed by Walters, Davidson, and Tuck which consists of soldering the wire to a thick coiled spring and twisting one end of the spring with respect to the other.[18] The titanium alloy used by Walters to fabricate the spring, which is difficult to solder to, has been replaced by a 2% beryllium doped copper alloy that has an elastic limit of about 0.9% at room temperature. Since soldering the sample to this material is easy, both compressive and tensile strain can be applied to the sample. The variable temperature environment is similar to that in $J_c(B,T)$ probes developed in our group.[19] The sample is in a vacuum chamber surrounded by a demountable can. Previously either solder or vacuum grease was used to attach the can to the probe. In the new probe, a copper gasket and knife edges, typically used in high vacuum systems, form the seal between the can and the probe. This seal is mechanically strong enough to sustain the torque applied to the spring. A thermometry block containing a calibrated Rh–Fe thermometer and a field-independent capacitance thermometer is located inside

[a]Electronic mail: najib.cheggour@durham.ac.uk

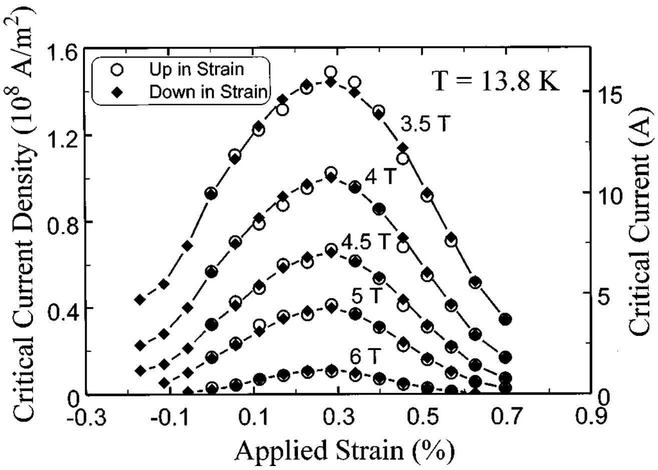

FIG. 1. $J_c(B,\epsilon)$ results at 13.8 K using the 1 μV/cm criterion, for a bronze processed Nb_3Sn multifilamentary wire. The results show an excellent reversibility of J_c with applied strain.

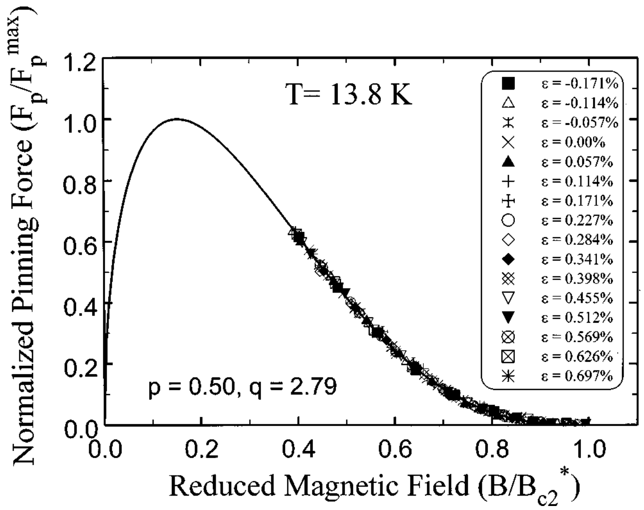

FIG. 2. Normalized pinning force as a function of reduced magnetic field at variable applied strain at 13.8 K, for a bronze processed Nb_3Sn multifilamentary wire. The continuous line represents a function proportional to $b^{0.50}(1-b)^{2.79}$.

the spring. We have improved the temperature accuracy over previous work[20] by placing a Cernox thermometer next to the sample and calibrating the Rh–Fe thermometer with respect to the temperature of the sample.

III. SAMPLE PREPARATION AND MEASUREMENTS

The wire used was a bronze processed Vacuumschmelze Nb_3Sn multifilamentary wire. It has a diameter of 0.37 mm. It contains 4500 Nb filaments embedded into CuSn alloy and is nonstabilized. The bronze to superconductor ratio is 2.4. The sample was wound on a stainless steel sample holder and heat treated under an argon atmosphere at 700 °C for 64 h. It was then carefully transferred onto the spring and soldered to it. $J_c(B,T)$ measurements were first taken at zero strain, from 6.5 to 16.5 K every 2.5 K. The strain was then incremented by $\Delta\epsilon=0.057\%$ up to $+0.7\%$. At each value of applied strain, J_c measurements were taken at 9 and 13.8 K as a function of magnetic field up to 15 T. While releasing the strain, measurements were taken at 13.8 K to check the reversibility of J_c. After returning to zero strain, compressive strain was then applied with the same incremental change down to -0.171% and measurements taken at 9 and 13.8 K.

IV. RESULTS

The J_c values are calculated for the overall wire cross section, using the standard 1 μV/cm criterion. The $J_c(B,\epsilon)$ results for 13.8 K are presented in Fig. 1. The shape of the curves is similar to results reported for 4.2 K.[8,9,11] At each field, J_c peaks at the same value of strain (ϵ_{max}), which is attributed to precompression exerted by the bronze matrix on the Nb_3Sn filaments as the sample is cooled down after the heat treatment. At 9 and 13.8 K, the maximum values of J_c are obtained for the same strain ϵ_{max}. The value of ϵ_{max} ($=0.257\%$) is in good agreement with that reported for similar Nb_3Sn specimens at 4.2 K in the VAMAS program report.[21] Figure 1 also shows that after straining the sample to a maximum tensile value of $+0.7\%$, J_c remains reversible.

To obtain $B^*_{c2}(T,\epsilon)$ values and describe the field dependence of F_p, Eq. (1) was used. By globally optimizing the fit to the data obtained at 13.8 K, a local shallow minimum at $p=0.50$ and $q=2.79$ was found. Figure 2 shows that at 13.8 K, a scaling law for F_p with reduced magnetic field accurately describes the data for all applied strain values used, whether compressive or tensile. The same values of p and q were used in fitting the variable strain data obtained at 9 K and the variable temperature data obtained at zero strain. The good agreement found using the fitting procedure demonstrates that F_p can be written as $F_p=K(T,\epsilon)b^{0.50}(1-b)^{2.79}$, where $K(T,\epsilon)$ is an arbitrary function of T and ϵ, although the values of p and q may not be considered unique. In Fig. 3, the normalized field $B^*_{c2}(T,\epsilon)/B^*_{c2\,max}(T)$ at 9 and 13.8 K is shown and compared to the results at 4.2 K from Ekin.[9] The peak in $B^*_{c2}(\epsilon)$ and $J_c(\epsilon)$ occurs at the same strain (ϵ_{max}). $B^*_{c2}(T,\epsilon)/B^*_{c2\,max}(T)$ is not a universal function

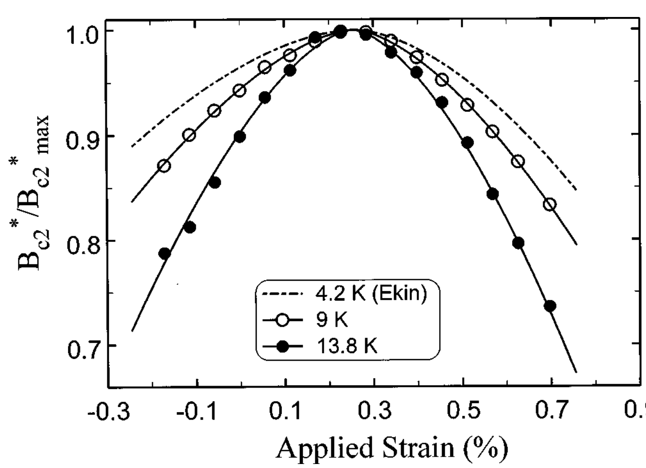

FIG. 3. Normalized field $B^*_{c2}(\epsilon)/B^*_{c2\,max}$ as a function of applied strain at 9 and 13.8 K, for a bronze processed Nb_3Sn multifilamentary wire. The dotted line shown represents Ekin's data at 4.2 K, for various Nb_3Sn specimens (see Ref. 9).

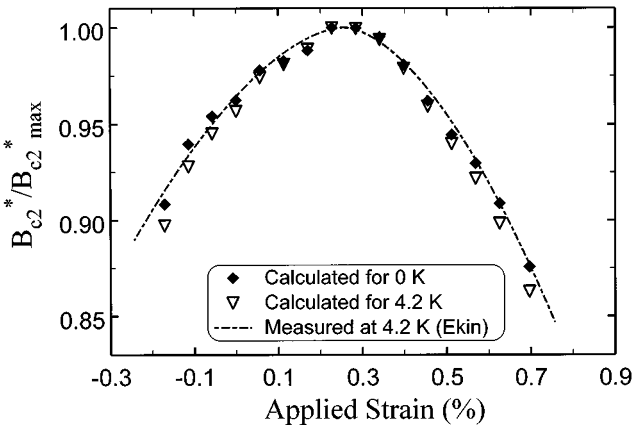

FIG. 4. $B^*_{c2}(\epsilon)/B^*_{c2\ \max}$ calculated values for 0 and 4.2 K, for a bronze processed Nb_3Sn multifilamentary wire. $B^*_{c2}(\epsilon)/B^*_{c2\ \max}$ deduced using the experimental results of $B^*_{c2}(\epsilon)$ obtained for 9 and 13.8 K and the expression $B^*_{c2}(T,\epsilon)=B^*_{c2}(0,\epsilon)[1-T/T^*_c(\epsilon)]$. $B^*_{c2}(\epsilon)/B^*_{c2\ \max}$ calculated values for 4.2 K are in a good agreement with Ekin's data (see Ref. 9).

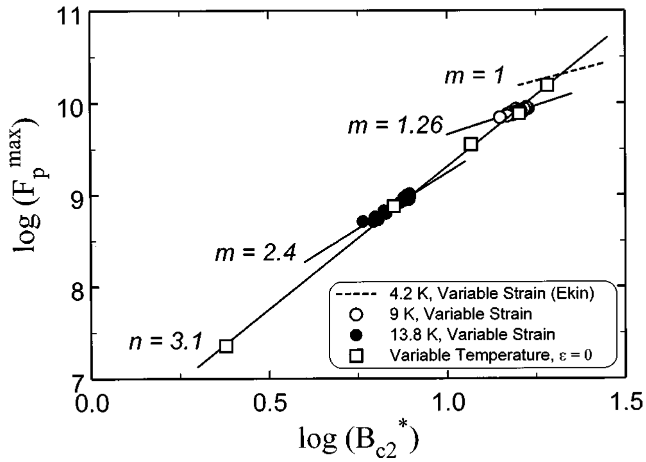

FIG. 5. Plots of $\log(F_p^{\max})$ vs $\log(B^*_{c2})$ for variable temperature and variable applied strain conditions, for a bronze processed Nb_3Sn multifilamentary wire.

of strain but is temperature dependent. The higher the temperature, the more sensitive $B^*_{c2}(T,\epsilon)/B^*_{c2\ \max}(T)$ is to strain. These results are in contrast to those of Kroeger *et al.*,[16] which showed no correlation between B^*_{c2} and J_c. Moreover, in their results, strain scaling was not observed.

V. TEMPERATURE AND STRAIN DEPENDENCE OF THE FIELD B^*_{c2}

Several studies of the Ginzburg–Landau theory have shown that the temperature dependence of the upper critical field $[B_{c2}(T)]$ can be described by $B_{c2}(T)=B_{c2}(0)g(T/T_c)$. Hence, we suggest that the strain and temperature dependence of B^*_{c2} can be expressed as

$$B^*_{c2}(T,\epsilon)=B^*_{c2}(0,\epsilon)g\left(\frac{T}{T^*_c(\epsilon)}\right), \tag{3}$$

where $g[T/T^*_c(\epsilon)]$ describes the temperature dependence of B^*_{c2} and also incorporates strain through the term $T^*_c(\epsilon)$, the temperature at which $B^*_{c2}(\epsilon)$ extrapolates to zero. To test experimentally the validity of Eq. (3), one can use the data obtained at 9 and 13.8 K to calculate $B^*_{c2}(4.2\,\mathrm{K},\epsilon)$ for comparison with Ekin's results. For a wide temperature range, our variable temperature data for the Nb_3Sn wire at zero strain show that to a good approximation, $B^*_{c2}(T)$ is fairly linear. Hence, the high temperature form of B^*_{c2} we suggest for the Nb_3Sn wire is

$$B^*_{c2}(T,\epsilon)=B^*_{c2}(0,\epsilon)\left(1-\frac{T}{T^*_c(\epsilon)}\right). \tag{4}$$

$T^*_c(\epsilon)$ is deduced from a linear extrapolation of B^*_{c2} at 9 and 13.8 K to $B^*_{c2}=0$. Although only 2 points were used to evaluate $T^*_c(\epsilon)$ for each strain, the shape of $T^*_c(\epsilon)$ is fairly smooth and has its peak value at the same strain as $B^*_{c2}(\epsilon)$ does. To calculate the absolute values for $B^*_{c2}(T,\epsilon)$ at low temperatures, we need a more general form of $g[T/T^*_c(\epsilon)]$ than a simple linear temperature dependence to incorporate the saturation of $B^*_{c2}(T)$ for low temperatures. However, in the low temperature regime, $T/T^*_c(\epsilon)$ tends to zero so that $g[T/T^*_c(\epsilon)]$ has a weak dependence on strain. Therefore, $B^*_{c2}(T,\epsilon)/B^*_{c2\ \max}(T)$ can be calculated quite accurately, independent of the exact form of g. Using Eq. (4), $B^*_{c2}(0,\epsilon)$ was determined and $B^*_{c2}(T,\epsilon)/B^*_{c2\ \max}(T)$ calculated for 4.2 and 0 K as shown in Fig. 4. The calculation for 4.2 K is in a good agreement with Ekin's results, which confirms the validity of Eq. (3). This equation shows that by knowing $T^*_c(\epsilon)$, $B^*_{c2}(0,\epsilon)$ and g, one can find $B^*_{c2}(T,\epsilon)$ for any temperature and strain provided that the applied strain keeps J_c in the reversible regime.

VI. UNIFIED SCALING LAW FOR THE PINNING FORCE DENSITY

In Fig. 5, $F_p^{\max}$ versus B^*_{c2} is shown for all strains and temperatures measured. Variable temperature data at zero strain give a value for n for the whole temperature range of 3.1. The variable strain data at fixed temperature show that if the data are parameterised using Eq. 2, m is a strong function of temperature. In contrast, the index n is strain independent and has an average value of 2.95 ± 0.15 in the temperature

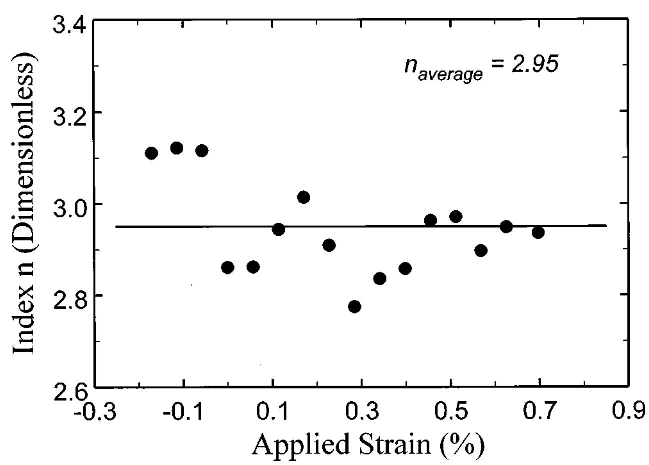

FIG. 6. The index n as a function of strain in the temperature range from 9 to 13.8 K. The index is defined by the equation $F_p=A(\epsilon)\times[B^*_{c2}(T,\epsilon)]^n b^{0.5}(1-b)^{2.79}$.

range from 9 to 13.8 K (Fig. 6). Hence, the $J_c(B,T,\epsilon)$ data presented in this work can be described by a unified temperature, strain, and magnetic field scaling law of the general form

$$F_p(B,T,\epsilon)=A(\epsilon)[B_{c2}^*(T,\epsilon)]^n b^p (1-b)^q, \tag{5}$$

where $n=2.95$, $p=0.50$, $q=2.79$, and $A(\epsilon)$ is a function of strain alone. $A(\epsilon)$ goes through a minimum value ($A_{\min}$) at $\epsilon_{\max}$ and varies by about 40% over the range of strain investigated in this work. $A(\epsilon)$ can be parametrized as a function of strain using an expression similar to those proposed previously to describe $J_c(\epsilon)$ and $B_{c2}^*(\epsilon)$,[8,9] of the general form

$$A(\epsilon)=A_{\min}[1+a|\epsilon-\epsilon_{\max}|^u], \tag{6}$$

where the fitting parameters are: $A_{\min}\cong 8.9\times 10^6\ \mathrm{Nm^{-3}T^{-2.95}}$, $u\cong 2.3$ and $a\cong 4.1\times 10^4$ for $\epsilon\leqslant\epsilon_{\max}$ and $a\cong 7.8\times 10^4$ for $\epsilon\geqslant\epsilon_{\max}$.

VII. DISCUSSION

In order to describe with a single equation the variable strain data at 4.2 K and the variable temperature data at zero strain in the literature, Ekin discussed whether the constant of proportionality (A) in the Fietz–Webb law depends on strain as $[B_{c2}^*(4.2\,\mathrm{K},\epsilon)]^{-(n-1)}$ for Nb_3Sn, so that m becomes equal to 1.[9,12] One may have expected A to be determined by the strain dependence of fundamental superconducting parameters {and described empirically using $[B_{c2}^*(4.2\,\mathrm{K},\epsilon)]^{-(n-1)}$} since at that time a wide range of Nb_3Sn conductors gave $m\sim 1$. The data presented here demonstrate that if variable strain data are parametrized using Eq. (2), the temperature dependence of the index m must be known for the conductor under investigation. Furthermore, recent variable strain measurements on Modified Jelly Roll Nb_3Sn wires at 4.2 K have shown that m changes from about 1.1 for a sample heat treated under atmospheric pressure to about 2.6 for a sample heat treated at high pressure.[22] This strongly suggests that $A(\epsilon)$ is not exclusively related to the fundamental superconducting parameters but also determined by extrinsic properties of the superconducting composite. The data of Kroeger *et al.* did not obey a scaling law of the form of Eq. (2). However they did find a prefactor similar to $A(\epsilon)$ and proposed its form may be due to a stress-induced martensitic transformation or the formation of twins in the sample.[16] Although the exact origin of $A(\epsilon)$ has yet to be clearly elucidated, it has to be stressed that $A(\epsilon)$ is a reversible function of strain alone and dependent on the material processing.

VIII. CONCLUSION

In summary, the temperature and strain dependence of B_{c2}^* has been described. Variable temperature, variable strain J_c data show that the reduced field dependence of F_p is broadly independent of temperature and strain, consistent with the Fietz–Webb and Ekin scaling laws. In order to eliminate the apparent inconsistency between these laws for the dependence of F_p on B_{c2}^*, a unified scaling law is proposed that describes the field, temperature, and strain dependence of F_p data. This law will contribute to the development of superconducting cryocooled high-field magnet technology.

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